ON THE DESIGN OF FLUIDIC FILTERS

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by
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CERTIFICATE

This is to certify that the work entitled 'ON THE DESIGN OF FLUIDIC FILTERS' is carried out by G.K. SHRIVASTAVA under my supervision and that it has not been submitted elsewhere for a degree.

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ABSTRACT

In this thesis, an investigation has been carried out on problem of fluidic filter design. The results on the design of fluidic filters have been put under two categories i) Lumped parameter fluidic filter design ii) Distributed parameter fluidic filter design. For lumped parameter fluidic filters, the concepts of modern electrical filter theory have been incorporated into the design techniques. For distributed parameter fluidic filters, a new design procedure has been evolved, based on certain basic results drawn from microwave filters.

The design of distributed parameter fluidic filters has been given greater attention since it involves a new kind of approach. Practical aspects of design have been studied. To simplify the job of fluidic filter design, a table of element values for fluidic filters has been constructed.

In presenting the results, an effort has been made to include, for its tutorial value, the basic results of electrical filter theory and fluidics. It has been done to make the work readable to electrical as well as mechanical engineers.

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SECTION 1
INTRODUCTION

1.1 FLUIDICS:

Fluidics is the technology using phenomenon of fluid flow in specially designed devices to perform functions of logic and control. Although fluidic systems can be studied in several different ways, the best approach, perhaps, is in terms of electrical analog.

Fluidics, as a technology, dates back to the year 1959 when a public announcement was made by the Harry Diamond Laboratories, an agency of US Army regarding development of fluid devices having no moving parts. Since then fluidics have been capturing the imagination of scientists and engineers greatly as the concept of 'no moving part' is a very attractive feature, apart from their reliability and economic considerations [1]. The newer commercial components not only have much better characteristics and less noise but they are also being built in modular form to minimise tubing and allow plug in facility.

The work in fluidics is mostly concentrated on devices like amplifiers, logic gates, diodes, triodes, modulators and bistable switches. Comparatively less attention has been given to the field of fluidic filters i.e. fluidic systems that discriminate between signals on the basis of harmonic components

In the past the filtering needs were generally met by electrical filters after converting the fluid signal into electrical signal by means of sensors. But of late it has been realised that filtering by fluidic means is a better proposition because of mainly two reasons [2]. Firstly, the electrical transducers have to be fitted at a distance from test point whic is generally inaccessible for such purposes. This requires the use of connecting tubing systems which introduce distortion. Secondly, where a large number of signals are involved, electrical filtering calls for elaborate circuitry which increases the overall cost significantly.

There has been some work on fluidic filters using lumped parameter approximation and transmission line effect but many ideas of modern electrical filter theory, still remain to be applied to this field. There is thus a good ground for investigation in this area with a view to extend some useful concepts of electrical filter theory to the field of fluidics. This thesis is concerned with such an investigation.

1.2 SCOPE OF WORK:

Fluidic filters, like their electrical counterparts can be broadly put in two categories, lumped parameter filters and distributed parameter filters. In lumped parameter theory, individual fluidic sections, depending upon their configurations,

are treated equivalent to lumped two terminal electrical elements. This is comparatively a simpler design technique, since it does not consider the phenomenon of wave propagation through the elements. This is more suitable if the overall dimensions of filter are smaller than quarter of the wavelength of signal propagating through it. Considerable work has been done in the development of this theory.

The distributed parameter theory considers the propagation of waves through the medium. It uses the distributive nature of parameters throughout a particular line section and the basic fluid elements, like microwave components, have 2-port configurations. Comparatively less attention has been given to this model. The results available in the area of fluidics consist essentially of heuristic techniques based on experimental observations and simulation. Systematic synthesis procedures, that yield filters to meet given specifications, are not yet available.

In the present work we are concerned with the synthesis of fluidic filters based on modern electrical filter theory. We have dealt with both the lumped and distributed parameters models. In lumped case we have essentially extended the concepts of modern filter theory to get a design procedure that meets the give filter specifications exactly. In distributed case, however we have applied the results of microwave

filter theory to fluidic filter design, leading to a design procedure that does not seem to have been available so far.

For the synthesis procedure for distributed parameter filters based on relavent features of microwave theory, we have chosen Richard's transformation and Kuroda's identity as tools for design of fluidic filters. The concept of unit element i.e., a lossless line section of fixed length, has been used. The procedure has been discussed in detail with the help of suitable examples.

We have also worked out the table of element values for fluidic filters of Butterworth kind. Butterworth filter configuration has been chosen as a sample case. Such tables can be obtained for other types of filters as well.

Our contribution is confined to the realisation part of the synthesis problem. Starting from the basic electrical design obtained from standard tables,we arrive at strueturcs of fluidic filters. In our examples we have mostly considered Butterworth approximation for simplicity. Other filter approximations like Chebychev, Bessel, elliptic etc. can be used when the situation demands.

Although fluidic and accoustic phenominon are closely connected by common principles, the literature on fluidic and acoustic filters seem to have grown independent of each

other. Here, we have investigated the work done in both the fields and presented them within a common frame-work. The theory outlined here is applicable to small amplitude fluidic signals which are basically of acoustic type. Hence under suitable assumptions, same theory can be applied to both fluidic and acoustic systems.

The present work is mostly confined to low pass filters. There are certain difficulties in high pass and band pass transformations in the fluidic context. These difficulties and the alternatives have been discussed in detail in the text.

Fluidics is a hybrid field requiring knowledge both of fluid mechanics and electrical engineering. No text is available which covers both the topics in sufficient detail. Therefore first few sections have been devoted to essential elements based on fundamentals drawn from both the fields. This has been done for the easy comprehension of both electrical as well as mechanical engineers.

To briefly summarise, an attempt has been made to cover the following points:

i. To propose a direct synthesis procedure for fluidic filters using modern filter theory.

ii. To study the fluidic and acoustic filters and try to correlate the two.

iii) To give a brief introduction to fluidics as well as modern electrical filter theory for proper understanding of fluidic filters.

This is primarily a theoretical investigation based on certain idealised forms of fluidic elements. We have, in particular, introduced the idea of fluidic filter design based on the lines of microwave filters. For the fullworth of this idea to be established, it would, of course, be necessary to study the practical behaviour of the proposed filters. But the time scale involved in practical studies is comparatively much longer and it was not possible to carry out such studies to a satisfactory degree in the available time.

1.3 ORGANISATION OF THESIS:

The thesis has been organised into seven sections as follows:

Section 2 deals with the fluidic transmission line theory. It gives the derivation of transmission line equations from the first principles. The basic fluid equations have been inspected to establish electrical analogy. We have, then, extended the electrical transmission line theory for the analysis of fluidic systems under various conditions. Use of system graphs have been made to depict the behaviour of fluidic elements diagramatically.

Section 3 is meant to familiarise mechanical engineers with essential aspects of electrical filter theory that are to be used here. Section 3.3 needs extra attention Since the proposed filter design is based on the theory outlined in this section.

Section 4 is more or less an investigation of the work done so far in the field of fluidic and acoustic filters. It is important in the context of lumped parameter filters since we have tried to extend the concepts of modern filter theory to the existing design techniques.

Section 5 gives the proposed design technique for distributed parameter filters using concepts of modern filter theory. The procedure is based on Richard's transformation. It has been discussed in detail with the help of suitable examples. The practical aspects of filter design and extention of this theory to high pass and band pass cases have been examined. Finally we have worked out tables of element values for fluidic filters of various kinds.

Section 6 gives the results of the efforts made in the direction of practical studies in relation to fluidic filters using Richard's transformation as proposed in Section 5.

The suggestions for future investigations have been given in Section 7.

Appendix 'A' contains the list of some important formulas. Appendix 'B' gives the attenuation characteristic of the Butterworth filters. Appendix 'C' contains the table of element values for electrical filters of Butterworth kind.

Appendix 'D' presents a pascal program which converts an electrical design into a fluidic design. A number of filter designs have been studied using this program as given in Appendix 'E'. These results have been used for interpretation in Section 5.3.

Appendix 'F' contains another pascal program which has been used to obtain the table of element values for fluidic filters from the standard table of electrical filters. One such table of element values for Butterworth filters is given in Appendix 'G'. This type of table makes the task of practical fluidic filter design very easy.

1.4 MOTIVATION FOR THE PRESENT WORK:

The objective with which we started our work was to develop filters for smoothing pneumatic pressures in wind tunnel testing [2]. The types of construction used were similar to those used in fluidic filters. Subsequently we went on to find that there was lot of similarity in the approaches for fluidic and acoustic filters. In fact there were instances of fluidic filters being used successfully for acoustic purposes. This led us to the more general problem of fluidic filter design Based on the applications of fluidic filters we have in mind, we have considered air as the medium of transmission in the

SECTION 2

FLUIDIC SYSTEM THEORY

Most of the fluidic devices, specially filters, work using processes of fluid flow through tubes and constrictions. The phenomenon of fluid flow through these elements is analysed based on the fluidic transmission line theory. The concept of transmission line has been adopted from electrical engineering based on the observation that the fundamental fluid equations are of the same form as those of the electrical transmission line.

An exact analysis of transmission line networks is based on distributed parameter theory. However to simplify the procedure and to reduce the complexity, lumped parameter technique is used with suitable assumptions.

In what follows, we have drawn an analogy between electrical and fluidic systems based on transmission line theory. Subsequently we have examined how fluidic systems can be analysed using this theory.

2.1 ELECTRICAL TRANSMISSION LINE THEORY:

The circuit model of a transmission line of infinitesimal length $\Delta \ell$ is shown in Fig. 2.1. The actual transmission line may be visualised as an infinite number of such sections connected in cascade.

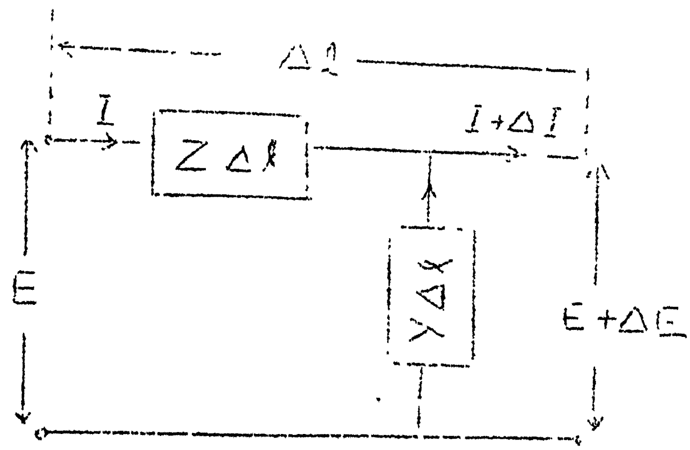


Figure 2.1: Circuit model of transmission line of infinitesimal length.

Z is the series impedance per unit length given by $Z = R + j\omega L$.

where R is resistence per unit length and

L is inductance per unit length.

Similarily Y is the shunt admittance per unit length given by $Y = G + j\omega C$.

where G is the conductance per unit length and

C is capacitance per unit length.

The expressions for the incremental voltage and current ΔE and ΔI can be found by applying Kirchoff's voltage and current law.

$$\Delta E = -IZ\Delta l \tag{2.1a}$$

and
$$\Delta I = -EY\Delta l \tag{2.1b}$$

In the limit as $\Delta \ell$ approaches zero equation (2.1) becomes

$$\frac{\partial E}{\partial \ell} = -ZI \tag{2.2a}$$

$$\frac{\partial I}{\partial \ell} = -YE \tag{2.2b}$$

Differentiating equation (2.2) with respect to ℓ, we get

$$\frac{\partial^2 E}{\partial \ell^2} = ZYE \tag{2.3a}$$

$$\frac{\partial^2 I}{\partial \ell^2} = ZYI \tag{2.3b}$$

Equations (2.3) are the fundamental equations of transmission lines [4]. They can be solved using conventional methods as given below:

In terms of operator, equation (2.3a) becomes

$$(m^2 - ZY) = 0$$

$$m = \pm\sqrt{ZY}$$

Hence the complete solution to equation (2.3a) will be

$$E = A\, e^{\sqrt{ZY}\ell} + B\, e^{-\sqrt{ZY}\ell} \tag{2.4a}$$

Equation (2.3b) can also be solved similarily

$$I = C\, e^{\sqrt{ZY}\ell} + D\, e^{-\sqrt{ZY}\ell} \tag{2.4b}$$

Equations (2.4) are the equations of a transmission line of length l. Let us predict the behaviour of a transmission line shown in Figure 2.2 with the help of these equations.

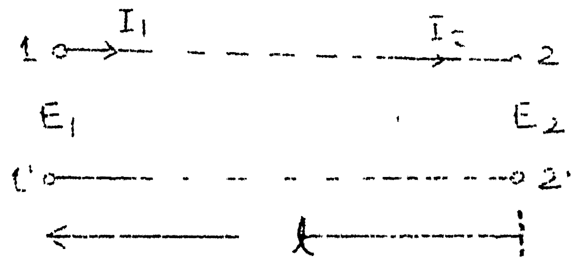


Figure 2.2: Transmission Line

The values of constants A,B,C and D in equations (2.4) can be found out by applying the boundary conditions. Since we are taking the distance l with reference to port 2, at $l = 0$, $E = E_2$ and $I = I_2$. From this, the values of A,B,C and D come out to be as given below.

$$A = \frac{E_2}{2} + \frac{I_2}{2}\sqrt{\frac{Z}{Y}}$$

$$B = \frac{E_2}{2} - \frac{I_2}{2}\sqrt{\frac{Z}{Y}}$$

$$C = \frac{I_2}{2} + \frac{E_2}{2}\sqrt{\frac{Z}{Y}}$$

$$D = \frac{I_2}{2} - \frac{E_2}{2}\sqrt{\frac{Z}{Y}}$$

By putting these values in equation (2.4) we get

$$E_1 = E_2\left(\frac{e^{\sqrt{ZY}l} + e^{-\sqrt{ZY}l}}{2}\right) + I_2\sqrt{\frac{Z}{Y}}\left(\frac{e^{\sqrt{ZY}l} - e^{-\sqrt{ZY}l}}{2}\right) \quad (2.5a)$$

$$I_1 = I_2 \left(\frac{e^{\sqrt{ZY}\ell} + e^{-\sqrt{ZY}\ell}}{2}\right) + (E_2/\sqrt{\tfrac{Z}{Y}})\left(\frac{e^{\sqrt{ZY}\ell} - e^{-\sqrt{ZY}\ell}}{2}\right) \quad (2.5b)$$

From equation (2.5), the input impedance, Z_c of a transmission line of infinite length can be found out to be

$$Z_c = \lim_{\ell \to \infty} \frac{E_1}{I_1} = \sqrt{\frac{Z}{Y}}$$

The input impedance Z_c of a transmission line of infinite length signifies a fundamental property of a transmission line. It is known as characteristic impedance.

Another important property of a transmission line is the propagation constant Γ given by

$$\Gamma = \sqrt{ZY} = \alpha + j\beta$$

where α is the attenuation constant and

β is the phase constant.

Using these notations, the transmission line equations can be written in following form.

$$E_1 = E_2 \cosh\Gamma\ell + I_2 Z_c \sinh\Gamma\ell \quad (2.6a)$$

$$I_1 = I_2 \cosh\Gamma\ell + \frac{E_2}{Z_c} \sinh\Gamma\ell \quad (2.6b)$$

These equations can be expressed in matrix form as given in (2.7).

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh\Gamma\ell & Z_c\sinh\Gamma\ell \\ \frac{1}{Z_c}\sinh\Gamma\ell & \cosh\Gamma\ell \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix} \quad (2.7)$$

The input impedance of a transmission line terminated in an impedance Z_L can be found from equation (2.7) as given below.

$$Z_{in} = \frac{E_1}{I_1} = Z_c \frac{Z_L \cosh \Gamma \ell + Z_c \sinh \Gamma \ell}{Z_L \sinh \Gamma \ell + Z_c \cosh \Gamma \ell} \qquad (2.8)$$

2.2 BASIC FLUID EQUATIONS:

Let us now examine the basic fluid equations and see whether they bear any analogy with the electrical transmission line equations [5]. The exact treatment of fluid equations is very complex and unweildy . The equations, however, acquire a substantially simplified form if we impose on them certain additional conditions that hold good in majority of cases of practical interest. There are 5 such basic conditions.

a) Laminar flow

b) Axial symmetry. There is no tangential velocity component.

c) Small amplitude signals. The density variations are small compared to average density. The signals are therefore, of acoustic type.

d) Small viscous forces, so that rate of change of axial velocity with distance is small.

e) Radius or width of tube is small compared to the wavelength. This means that pressure is uniform over the cross section and that radial velocity is zero.

2.2.1 Continuity Equation:

Consider the flow of fluid through an arbitrary fixed, closed surface of area A and volume V as given in Figure 2.3.

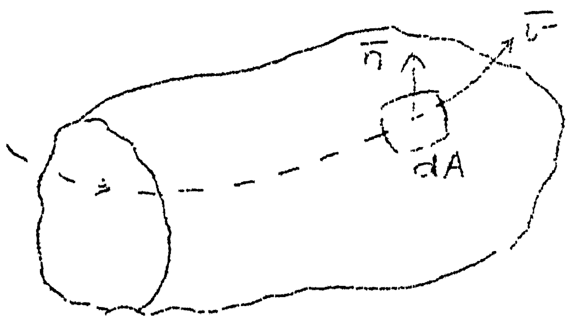


Figure 2.3:An arbitrary fluid volume

From the law of conservation of mass, the rate of change of mass must be equal to the rate at which fluid is entering the closed surface.

$$\int_V \frac{\partial \rho}{\partial t} \, dV = - \int_A \rho \, \bar{v} \cdot \bar{n} \, dA \tag{2.9}$$

where ρ = fluid density

V = volume

A = area

From Gauss' theorem

$$\int_A \rho \, \bar{v} \cdot \bar{n} \, dA = \int_V \nabla \cdot \rho \, \bar{v} \, dV$$

Equation (2.9) therefore can be written as

$$\int_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \, \bar{v} \right) dV = 0$$

Since control volume dV is of arbitrary size

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \bar{v} = 0 \tag{2.10}$$

As we are considering the flow of fluid through tubes, it is more convenient to use cylindrical coordinates. Equation (2.10), therefore become

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho v_r r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho v_\psi)}{\partial \psi} + \frac{\partial(\rho v_x)}{\partial x} = 0$$

where r = radius

v_r = radial velocity

v_ψ = tangential velocity

v_x = axial velocity

But we have assumed radial and tangential velocities (v_r and v_ψ) to be zero. The continuity equation, therefore reduces to

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v_x}{\partial x} = 0$$

or

$$\frac{\partial \rho}{\partial p} \frac{\partial p}{\partial t} = - \frac{\partial v_x}{\partial x}$$

where p is the pressure

But $\frac{\partial \rho}{\partial p} = \frac{1}{a^2}$ where a is speed of sound. The continuity equation, therefore, reduces to

$$\frac{\partial v_x}{\partial x} = - \frac{1}{a^2} \frac{\partial p}{\partial t} \tag{2.11}$$

2.2.2 Momentum Equation:

From Newton's second law, the force per unit volume F is equal to the rate of change of momentum of fluid. Therefore

$$F = \int_V \frac{\partial \rho \bar{v}}{\partial t} dV + \int_A \rho \bar{v} (\bar{v}.\bar{n}) dA \qquad (2.12)$$

By Gauss' Theorem

$$\int_A \rho \bar{v}(\bar{v}.\bar{n}) dA = \int_V [\rho \bar{v}(\nabla . \bar{v}) + (\bar{v}.\nabla)\rho \bar{v}] dV$$

equation (2.12), therefore, becomes

$$F = \int_V [\frac{\partial \rho \bar{v}}{\partial t} + \rho \bar{v}(\nabla . \bar{v}) + (\bar{v}.\nabla)\rho \bar{v}] dV$$

But $\frac{\partial \rho \bar{v}}{dt} + (\bar{v}.\nabla) \bar{v} = \frac{d \rho \bar{v}}{dt} = \rho \frac{d\bar{v}}{dt} + \bar{v} \frac{d\rho}{dt}$

$$\therefore \quad F = \int_V (\rho \frac{d\bar{v}}{dt} + \bar{v} [\frac{d\rho}{dt} + \rho (\nabla . \bar{v})]) dV \qquad (2.13)$$

Again from eqn. (2.10)

$$\frac{\partial \rho}{\partial t} + \nabla . \rho \bar{v} = 0$$

or $\frac{\partial \rho}{dt} + \rho \nabla . \bar{v} + \bar{v}.\nabla \rho = 0$

or $\frac{d\rho}{dt} + \rho \nabla . \bar{v} = 0$

Therefore bracketed term in equation (2.13) becomes zero.

or $$F = \int_V [\rho \frac{d\bar{v}}{dt}] dV \qquad (2.14)$$

But F itself is given by

$$F = \int_V (\rho f + R)\, dV$$

where f is the body force and R is the viscous loss.

Equation (2.14) therefore gives

$$\rho \frac{d\bar{v}}{dt} = \rho f + R$$

Considering cylindrical coordinates, the momentum equation becomes

$$\left(\frac{\partial v_x}{dt} + v_r \frac{\partial v_x}{\partial r} + v_x \frac{\partial v_x}{\partial x} \right) = - \frac{\partial p}{\partial x}$$

$$+ \mu \frac{\partial}{\partial x} \left[- \frac{2}{3r} \frac{\partial}{\partial r}(r v_r) + \frac{4}{3} \frac{\partial v_x}{\partial x} \right]$$

$$+ \frac{\mu}{r} \frac{\partial}{\partial r} r \left(\frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x} \right) \qquad (2.15)$$

Since we have already assumed that the radial velocity is zero, the equation (2.15) reduces to

$$\frac{\partial v_x}{\partial t} = - \frac{\partial p}{\partial x} + \frac{4}{3} \mu \frac{\partial^2 v_x}{\partial x^2} + \frac{\mu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_x}{\partial r} \right) \qquad (2.16)$$

Since we have assumed that rate of change of axial velocity with distance is small, the term $\frac{\partial^2 v_x}{\partial x^2}$ can be neglected. Equation (2.16), therefore, becomes

$$\frac{\partial v_x}{\partial t} = - \frac{\partial p}{\partial x} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_x}{\partial r} \right) \right] \qquad (2.17)$$

2.3 PROPAGATION MODEL FOR FLUIDIC LINES:

Based on the fluid equations derived in the preceding section, we will derive two simple electrical models of fluidic lines [1]. They are i) lossless line model and ii) average friction model. There is slight difference in the treatment of average friction model for circular and rectangular lines. Therefore, they have been discussed separately.

2.3.1 Lossless Line Model:

Let us consider a section of fluidic line of infinitesimal length as shown in Figure 2.4.

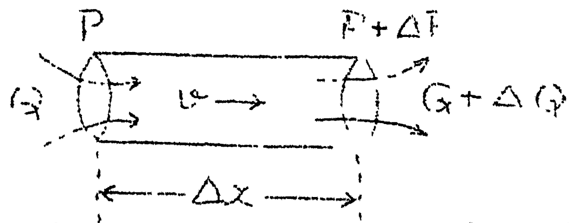


Figure 2.4: A fluidic line of infinitesimal length

For the analysis of lossless fluidic line we assume that the effect of viscousity can be neglected. The continuity and momentum equations, then, become as follows.

$$\frac{\partial p}{\partial x} = - \rho \frac{\partial v}{\partial t} \tag{2.18a}$$

$$\frac{\partial v}{\partial x} = - \frac{1}{\rho a^2} \frac{\partial p}{\partial t} \tag{2.18b}$$

Equations (2.18) can be written in terms of volume flow $q = Av$, where A is the cross-sectional area.

$$\frac{\partial p}{\partial x} = -\frac{\rho}{A}\frac{\partial q}{\partial t} \tag{2.19a}$$

$$\frac{\partial q}{\partial x} = -\frac{A}{\rho a^2}\frac{\partial p}{\partial t} \tag{2.19b}$$

Taking Laplace transform

$$\frac{\partial P}{\partial x} = -\left(\frac{\rho S}{A}\right) Q \tag{2.20a}$$

$$\frac{\partial Q}{\partial x} = -\left(\frac{AS}{\rho a^2}\right) P \tag{2.20b}$$

where $S = j\omega$

Here we see that equations(2.20) are of the same form as the equations (2.2) of electrical transmission line. Now there are two possible ways of describing the electrical analog. One way of describing results from identifying P with E and Q with I i.e. $P \equiv E$ and $Q \equiv I$; the other dual way results from identifying P with I and Q with E i.e. $P \equiv I$ and $Q \equiv E$. From a purely mathematical point of view, it does not make any difference as to which analogy we choose. However, from the point of view of the physical phenomenon of flow of fluid through lines, the two lead to different interpretations for the physical variables.

To see this, let us consider a junction of fluidic lines given in Figure 2.5. We can say that the vector sum of mass flow is zero at the junction. This statement is equivalent to Kirchoff's current law in electrical engineering

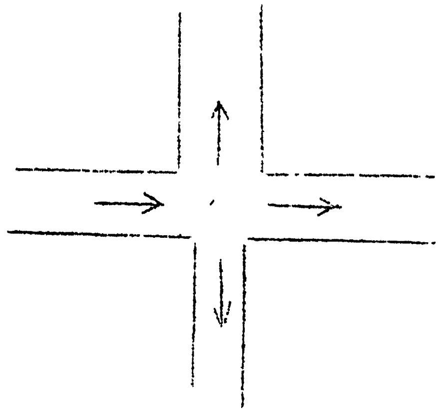


Figure 2.5: Junction of a fluidic line

if we say $Q \equiv I$. But if we consider $Q \equiv E$ then electrical equivalent becomes the Kirchoff's voltage law which is concerned with voltages in a loop.

Since $Q \equiv I$ allows us to think of a fluidic junction in terms of an electrical junction rather than a mesh, most traditional treatments adopt this analogy. With this analogy we can represent the series impedance and shunt admittance per unit length of a fluidic lines as

$$Z = \frac{\rho s}{a} = j\omega \frac{\rho}{A} \tag{2.21a}$$

$$Y = \frac{As}{\rho a^2} = j\omega \frac{A}{\rho a^2} \tag{2.21b}$$

Thus we see that by neglecting the effect of viscosity, a fluidic line can be expressed in terms of electrical transmission line with

$$R = G = 0$$

$$L = \frac{\rho}{A} \tag{2.22a}$$

$$C = \frac{A}{\rho a^2} \tag{2.22b}$$

These are the parameters of a lossless electrical transmission line and accordingly we call this model as lossless fluidic line model. The characteristic impedance Z_c and propagation constant Γ of this line are given as

$$Z_c = \sqrt{\frac{Z}{Y}} = \frac{\rho a}{A} \tag{2.23}$$

$$\Gamma = \sqrt{ZY} = j\,\frac{\omega}{a} = j\beta \tag{2.24}$$

We see, here, that Z_c is purely resistive and Γ has imaginary value.

2.3.2 Average Friction Model for Circular Lines:

In the preceeding paragraphs we saw that a fluidic line can be treated as a lossless transmission line if the effect of viscousity is neglected. Yet another way of treating the fluidic line is by considering the frictional losses due to viscousity. This is a more appropriate method since certain amount of loss is always associated with the flow of fluid in the tubes. We will first consider a circular fluidic line with r as the radius. By taking the average over the cross-section area of tube, equation (2.17) can be written as

$$\frac{\partial p}{\partial x} = -\rho\,\frac{\partial v}{\partial t} + \frac{2\pi\mu}{A}\int_0^{r_w} \frac{\partial}{\partial r}\left(r\,\frac{\partial v}{\partial r}\right)dr$$

or

$$\frac{\partial p}{\partial x} = -\rho\,\frac{\partial v}{\partial t} + \frac{2\pi\mu r_w}{A}\left(\frac{\partial v}{\partial r}\right)_{r=r_w} \tag{2.25}$$

To evaluate the velocity gradient, we assume a velocity distribution $v(r) = 2v\,(1 - \frac{r^2}{r_w^2})$

$$\left(\frac{\partial v}{\partial r}\right)_{r=r_w} = -\frac{4v}{r_w}$$

Equation (2.25) then becomes

$$\frac{\partial p}{\partial x} = -\left(\rho\,\frac{\partial v}{\partial t} + \frac{8\pi\mu v}{A}\right)$$

Replacing average velocity $v = \frac{q}{A}$ and taking laplace transform, we get

$$\frac{\partial p}{\partial x} = -\left(\frac{\rho s}{A} + \frac{8\pi\mu}{A^2}\right)Q \qquad (2.26)$$

The continuity equation remains same as that for lossless case i.e.

$$\frac{\partial Q}{\partial x} = -\left(\frac{As}{\rho a^2}\right)P \qquad (2.27)$$

Comparing equations (2.26) and (2.27) with equations (2.2), we get

$$Z = \frac{\rho s}{A} + \frac{8\pi\mu}{A^2} \qquad (2.28a)$$

$$Y = \frac{A_s}{\rho a^2} \qquad (2.28b)$$

The equation (2.28) of average friction case differs from the equations for lossless model by the term $\frac{8\pi\mu}{A^2}$ which

can be treated as the resistive portion of the series impedance.

$$R = \frac{8\pi\mu}{A^2} \tag{2.29}$$

2.3.3 Average Friction Model for Rectangular Lines:

For finding out the equivalent resistence for the fluidic line with rectangular cross-section, we will have to consider the momentum equation for the rectangular coordinates i.e.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) \tag{2.30a}$$

Since we have assumed the radial and tangential velocities to be zero, the continuity equation will remain same i.e.

$$\frac{\partial u}{\partial x} = -\left(\frac{1}{\rho a^2}\right)\frac{\partial p}{\partial t} \tag{2.30b}$$

A close examination of equations (2.30) reveals that the treatment of rectangular fluid lines differs from that of circular fluidic line only by the viscousity term which results into resistive portion of series impedance. The velocity distribution in a rectangular fluid line is very complex. The solution of equation (2.30) comes out in form of a series. We will therefore, give the result directly in terms of the equivalent resistence without going into the details of solution.[1]

$$R = \frac{12\mu\sigma/A^2}{1 - \frac{192}{\pi^5}\ \frac{1}{\sigma}[\tanh \frac{\pi\sigma}{2} + \frac{1}{3^5}\ \tanh\ \frac{3\pi\sigma}{2} + \ldots\ ]}$$

This expression for the equivalent resistence is quite unwieldy and hence an approximate relation given below is often used.

$$R = \frac{12\mu\sigma}{A^2}\ (\ 1 + \frac{1}{\sigma^2}\ ) \qquad (2.31)$$

The values of L and C of rectangular fluid lines remain same as that for lossless model.

2.4 FLUIDIC TRANSMISSION LINE THEORY:

The fluidic lines can be represented and analysed in two ways. The first one is the distributed parameter method which uses the concept of distributive nature of parameters as discussed in connection with electrical transmission lines. However when the dimensions of a fluidic line are very small, the distributed effect of parameters can be lumped. Interconnection of such lines can be analysed on the basis of theory of lumped networks.

2.4.1 Distributed Parameter Theory:

In the preceeding section, we have seen that there is an analogy between the fluidic line equations and electrical

transmission line equations. Hence we can apply the same mathematical treatment to fluid lines what we applied for transmission lines. Therefore the behaviour of a typical fluidic line shown in Figure 2.6, can be predicted by using following matrix equation.

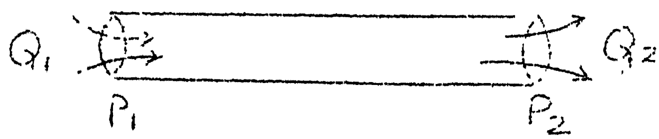


Figure 2.6: A typical fluidic line

$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} \cosh \Gamma l & Z_c \sinh \Gamma l \\ \frac{1}{Z_c} \sinh \Gamma l & \cosh \Gamma l \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix} \qquad (2.32)$$

A more general way of expressing a fluidic line is with the help of average friction model. This model gives the values of Z and Y as reproduced below.

$$Z = R + j\omega L$$

$$Y = j\omega C$$

where the values of fluidic resistence, inductance and capacitance, for a fluidic line of length l, are given as follows.

$$R = \frac{8\pi\mu}{A^2} l \text{ (for circular lines)}$$

$$L = \frac{\rho}{A} \ell$$

$$C = \frac{A}{\rho a^2} \ell$$

The characteristic impedance and propagation constants are obtained from these equations as given below.

$$Z_c = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{R+SL}{SC}}$$

$$= \sqrt{\frac{L}{C}} \sqrt{(1+\frac{R}{SL})} \qquad (2.33)$$

and,

$$\Gamma = \sqrt{ZY}$$

$$= \sqrt{(R+SL)SC}$$

$$= S\sqrt{LC} \sqrt{(1+\frac{R}{SL})} \qquad (2.34)$$

The characteristic impedance and propagation constant so obtained are complex functions of frequency. Handling of such expressions become very difficult specially when we are thinking in terms of synthesis of a filter.

Various scientists have given different empirical formulas to approximate the behaviour of lossy fluidic line. The simplest way of dealing the lossy fluidic line is to treat it as a lossless line with a resistence (signifying loss) in series. This assumption is valid only when the magnitude of the loss is very small compared to the reactive impedance.

The situation is very favourable when the resistence forms an extremely small part of reactance i.e. $R << \Omega L$. In that case we revert back to the simple model of lossless transmission line.

2.4.2 Lumped Parameter Theory:

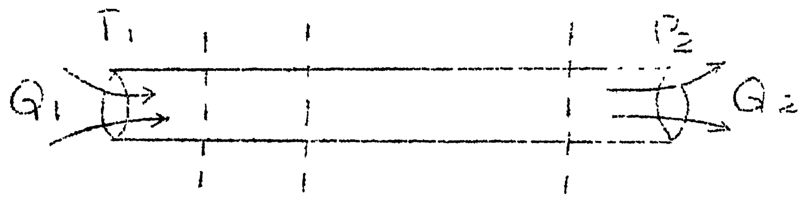
So far we have considered the analysis of fluidic lines based on distributed nature of parameters. This is rather complex since it involves hyperbolic functions. The analysis can be greatly simplified for a lossless line if we divide the fluidic line into small sections and treat the parameters as lumped for each individual section. We are focusing our attention to lossless case since for most of the practical applications, the fluidic lines can be expressed as a lossless transmission line.

In terms of electrical analogy, if we represent each section of a fluidic line by the transmission line model of Fig. 2.1, the overall equivalent circuit of a fluidic line can be expressed in form of a ladder network shown in Fig. 2.7.

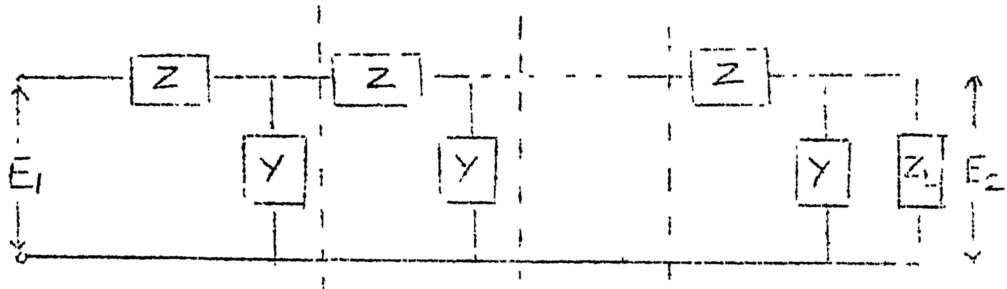
The series impedance and shunt admittance Z and Y are given by the following expressions.

$$Z = j\omega L = j\omega \left(\frac{\rho}{A}\ell\right)$$

$$Y = j\omega C = j\omega \left(\frac{A}{\rho a^2}\ell\right)$$



(a) Fluidic Line



(b) Equivalent electrical circuit

Figure 2.7: Equivalent electrical circuit of a lumped fluidic line.

where ℓ is the length of each lump i.e. each individual section of line.

In addition to providing a simple method of analysis of the lumped network model of figure 2.7, the lumped parameter theory also suggests a method of designing a fluidic system using lumped fluidic elements [6]. The key to this method is the behaviour of fluidic elements under certain limiting conditions relating the physical dimensions. We will study this behaviour in relation to circular fluidic lines which are commonly used.

Let us examine the matrix equation of a fluidic line

given by equation (2.32) and see how can we obtain the lumped circuit elements from the fluidic lines.

As a first step let us approximate the hyperbolic functions involved in the matrix equation. Thus sinhx and coshx can be expanded in a series form as given below.

$$\sinh x = x + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \ldots$$

$$\cosh x = 1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \ldots$$

If x is small enough so that the terms of second and higher order can be neglected, then sinh and cosh become

$$\sinh x \approx x$$

$$\cosh x \approx 1$$

In fluidic line equations the two hyperbolic functions we come across are $\sinh\Gamma\ell$ and $\cosh\Gamma\ell$ as shown below

$$\begin{bmatrix} P_1 \\ \\ Q_1 \end{bmatrix} = \begin{bmatrix} \cosh\Gamma\ell & Z_c\sinh\Gamma\ell \\ \\ \frac{1}{Z_c}\sinh\Gamma\ell & \cosh\Gamma\ell \end{bmatrix} \begin{bmatrix} P_2 \\ \\ Q_2 \end{bmatrix}$$

For a lossless fluidic line, the function $\Gamma\ell$ can be simplified as follows.

$$\Gamma\ell = j\beta\ell = j\,\frac{\omega}{a}\ell$$

If ℓ is very small, typically of the order of few mms, the condition $\Gamma\ell \ll 1$ is satisfied for all practical

purposes. Hence $\sinh \Gamma \ell$ can be replaced by $\Gamma \ell$ and cosh can be replaced by 1 without causing any significant error.

The matrix equation of a fluidic line then reduces to

$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_c \; j\beta\ell \\ \frac{1}{Z_c} \; j\beta\ell & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix} \qquad (2.35)$$

We also know from equation (2.23) that

$$Z_c = \frac{\rho a}{A}$$

Now there are two possibilities that can be encountered in practice. A fluidic line with sufficiently large diameter will lead to very small value of Z_c almost leading to zero. Thus in the limit $Z_c \to 0$ the matrix equation of equation (2.35) reduces to following form

$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_c} \; j\beta\ell & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}$$

But $\frac{1}{Z_c} \; j\beta\ell = j \, \frac{\omega}{a} \cdot \ell \cdot \frac{A}{\rho a}$

$$= j\omega\left(\frac{A}{\rho a^2}\right) \ell$$

By replacing $(\frac{A}{\rho a^2} l)$ by C, the capacitance in terms of electrical analog , from equation (2.22), the matrix equation becomes

$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix} \qquad (2.36)$$

This is of the some farm as the equation of a shunt capacitance as shown in Figure 2.8. This fact can be verified

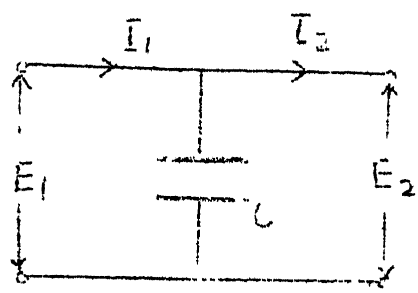
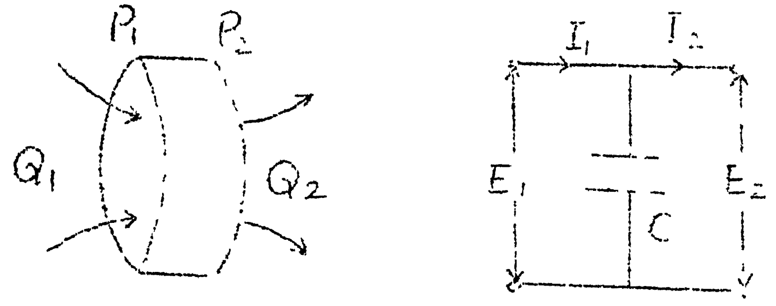


Figure 2.8: Electrical network involving a shunt capacitance.

by writing current and voltage equations for the network of Figure 2.8 which are

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} E_2 \\ I_2 \end{bmatrix}$$

Thus a fluidic line of small length and large diameter can be treated analogous to a shunt capacitance in an electrical network as shown in Figure 2.9.



(a) A fluidic line of small length and large diameter

(b) The electrical analog of the fluidic line

Figure 2.9: Electrical analog of a fluidic line of large diameter.

Another limiting condition encountered in the fluidic lines is the case of a line with very small diameter . In the limit $Z_c = \frac{\rho a}{A} \rightarrow \infty$ we can say that $\frac{1}{Z_c} \rightarrow 0$. The matrix equation then reduces to

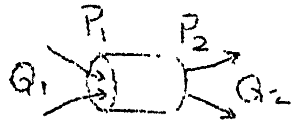
$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_c j\beta \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}$$

Proceeding in the same way as we did for the large diameter case, the above equation can be reduced to following form

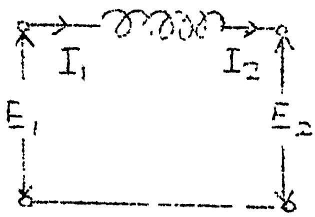
$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix} \qquad (2.37)$$

where $L = \frac{\rho}{A} \ell$

The fluidic line with small diameter, therefore, can be treated as analogous to series inductance in an electrical network as shown in Figure 2.10.



(a) A fluidic line with small diameter.



(b) The electrical analog of the fluidic line.

Figure 2.10 : Electrical analog of a fluidic line of small diameter.

The lumped parameter representation of the fluidic lines is of great importance in filter design. The low pass electrical filter networks are in the form of ladder networks comprising of series inductances and shunt capacitances. With the help of the fluidic line configurations discussed above an electrical filter design can be straightaway transformed into a fludic filter design.

2.5 SYSTEM GRAPH REPRESENTATION OF FLUIDIC ELEMENTS:

In lumped electrical system the physical circuit diagram and the theoretical network model are topologically the

same. The situation is not the same in fluidic systems where the physical arrangement does not directly give any indication of the inter-relation between the various variables and the equations governing them. The interdependence between the variables can be visualised in a much easier way with the help of what is called a system graph [7,8].

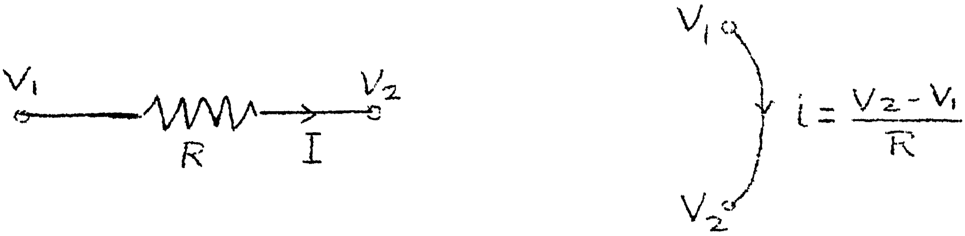
The system graph is the graphical representation of equations of a dynamic system in a manner that is inspired by the theory of multiterminal electrical networks. The system in this representation, is visualised as an interconnection between multiterminal elements. Each element is characterised by a constitutive relationship between what are called its through and across variables.

The type of elements we are going to discuss fall under two categories, two terminal elements and two port elements. A two terminal element is described by one through variable and one across variable say i and v respectively. The two are generally treated to be related in the form

$$v = f(i)$$

or equivalently $i = g(v)$

These equations are called the constitutive relationship of the two terminal element. We are concerned here with these constitutive relation. The lumped electrical resistance,

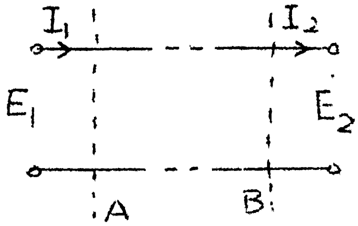


(a) Resistence

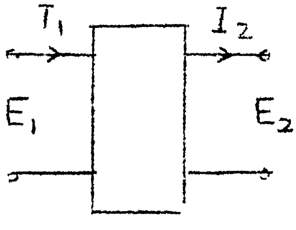
(b) Inductance

(c) Capacitance

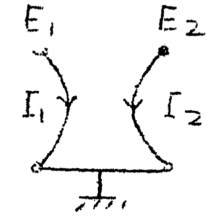
Figure 2.11: System graph of lumped electrical elements



(a) Transmission line



(b) Two port representation.



(c) System graph

Figure 2.12: System graph representation of a transmission

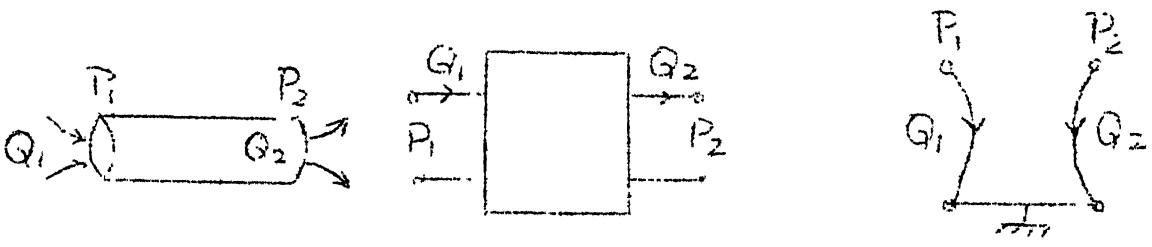
inductance and capacitance are typical examples of two terminal elements. In all electrical cases the system graph is simply the usual lumped network theory model of the element as shown in Figure 2.11.

A section of distributed parameter line, when considered as a single entity between the two ends, can be treated as two port element. The transmission line is an example of this. The system graph of a transmission line between the two ends 1 and 2 is shown in Figure 2.12.

2.5.1 Representation of Distributed Parameter Fluidic elements:

A fluidic line, like electrical transmission line can also be treated as a two port element between the two ends. The two set of variables are P_1, Q_1 at one port and P_2, Q_2 at the other port.

In the electrical system, voltages are taken with respec to ground, hence the system graph of a two port element has a common ground at both the ports. In the fluidic system the ground is analogous to atmospheric pressure. The system graph of a fluidic line is given in Figure 2.13.



(a) Fluidic line (b) Two port representation (c) System graph

We will now examine the behaviour of certain specific types of fluidic line configurations with the help of system graphs. The configurations of our primary interest are closed and open ended fluidic lines which will be used subsequently in the design of fluidic filters. Let us start with the matrix equation of a fluidic line given below.

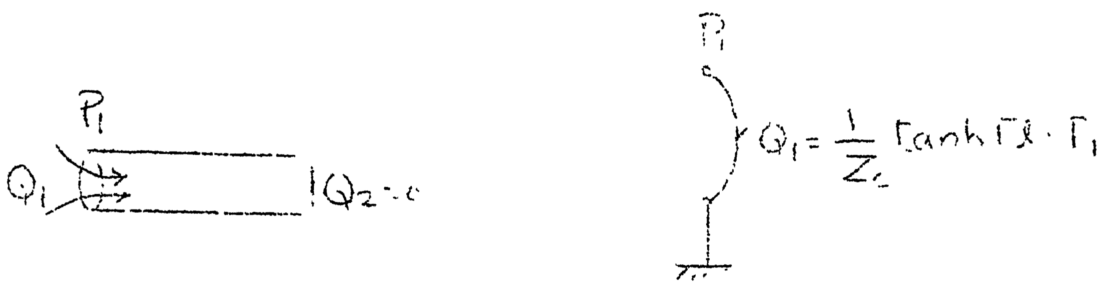
$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} \cosh\Gamma l & Z_c \sinh\Gamma l \\ \frac{1}{Z_c}\sinh\Gamma l & \cosh\Gamma l \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}$$

The closed ended fluidic line will offer an infinite impedance to the flow of fluid. Therefore $Q_2 = 0$. The system matrix then gives

$$\begin{aligned} P_1 &= \cosh\Gamma l \cdot P_2 \\ Q_1 &= \frac{\sinh\Gamma l}{Z_c} P_2 \\ &= \frac{1}{Z_c} \tanh\Gamma l \cdot P_1 \end{aligned} \qquad (2.38)$$

Thus the second port gets isolated. The constitutive relationship at the input can be given by the system graph of a two terminal element with one terminal forced to ground as shown in Figure 2.14.

The open ended line offers zero resistence to flow and hence $P_2 = 0$. The system matrix then gives the relations between the variables as given below.

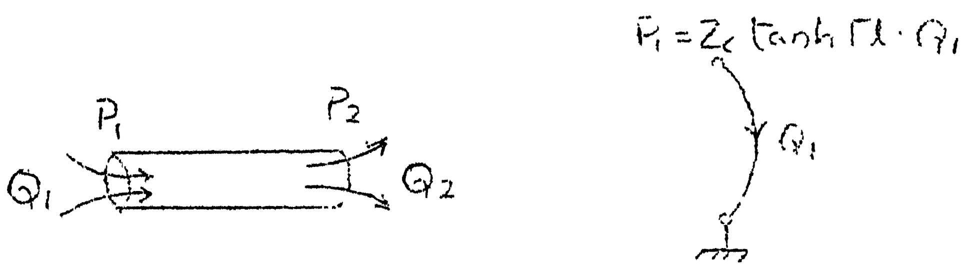


(a) Fluidic line (b) System graph

Figure 2.14: System graph of a closed ended fludic line.

$$
\begin{aligned}
Q_1 &= \cosh\Gamma l \,.\, Q_2 \\
P_1 &= Z_c \sinh\Gamma l \,.\, Q_2 \\
&= Z_c \tanh\Gamma l \,.\, Q_1 \qquad (2.39)
\end{aligned}
$$

Thus we see that the relation amongs the variables at the input of an open ended line can also be given by a two terminal graph as shown in Figure 2.15.



(a) Fluidic line (b) System graph

Figure 2.15: System graph representation of an open ended fluidic line.

In the above paragraphs, we have seen the two terminal behaviour of the closed and open ended fluidic lines. It may be

appreciated that in a composite fluidic system these fluidic lines can be only used at stub lines since the loading at the far end is prespecified. When used as stub lines, appropriate fluidic line sections will have to be used for the interconnection. Therefore, in stub line configuration, the closed and open ended fluidic lines shown in Figure 2.16 will behave like two port elements.

(a) Closed ended fluidic line (b) Open ended fluidic line.

Figure 2.16: Closed and opend ended lines used as stubs

To examin the behaviour of stub line configurations, we will use the following relations.

$$P_1 = P_2 = P$$

$$Q = Q_1 - Q_2$$

$$Q = aP$$

where $a = \frac{1}{Z_c} \tanh \Gamma \ell$ for closed line and

$a = \frac{1}{Z_c \tanh \Gamma \ell}$ for open line from equations (2.38) and (2.39).

These relations indicate that the behaviour of stub lines will be governed by following matrix equation.

$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}$$

The system graph will, therefore, be as given in Figure 2.17.

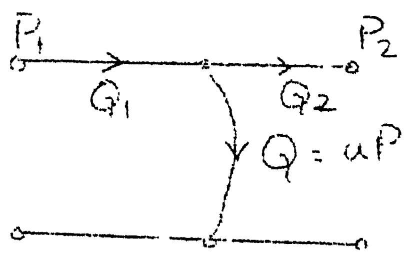
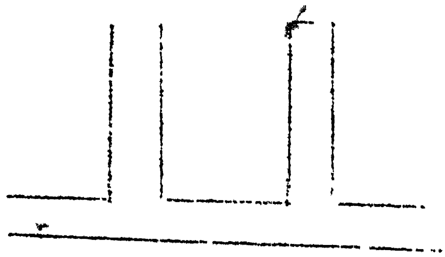


Figure 2.17: System graph of a closed or open ended line used as stub.

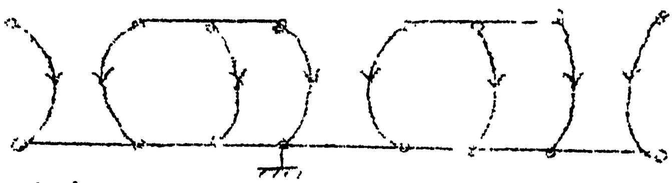
In both the cases of closed and open ended fluidic lines, the system graph will remain same, only the parameter 'a' will differ as given above.

The system graph of an overall fluidic system can be obtained by combining the system graphs of individual sections. To see how the system graphs are combined, consider the fluidic line elements placed in cascade as shown in Figure 2.18(a). Here the variables at the output of one element are the same as the variables at the input of next element. For the

purpose of mathematical analysis, this means that the matrix for the overall system is obtained by multiplying the system matrices of individual fluidic lines. Correspondingly the system graphs of elements appear in series with each other as shown in Figure 2.18(b).



(a) Fluidic line system



(b) System graph

Figure 2.18: System graph representation of a composite fluidic line structure.

2.5.2 Representation of Lumped Parameter Fluidic Elements:

We have seen in Section 2.4 that when the length of a fluidic line is very small, it can be treated as a lumped fluidic element, under certain limiting conditions. Let us now examine the behaviour of these elements and see whether system graphs support the electrical analogy which we have obtained based on the mathematical equations.

There are basically two types of lumped fluidic elements which we are going to use in the realisation of fluidic filters. The first one is a fluidic line with small cross sectional area which, in terms of electrical analog, behaves like an inductance. The other is the fluidic line with comparatively large cross-sectional area. It behaves like a capacitance in terms of electrical analog.

In the first case, i.e. a fluidic line of small length with a small cross-sectional area, the flow of fluid is almost direct from one port to other since the volume of the element is very small. This means $Q_1 = Q_2$. The relationship between the pressure variable can be obtained from the matrix equation (2.35).

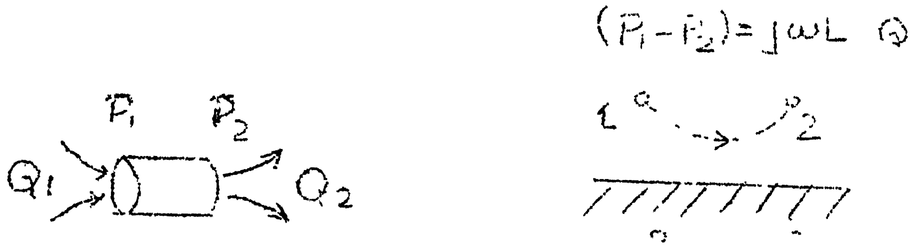
$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}$$

i.e. $Q_1 = Q_2 = Q$

$$P_1 - P_2 = j\omega L\ Q$$

where $L = \frac{\rho}{A} \ell$

The configuration of this element is basically of two terminal type. The system graph is given in Figure 2.19.



(a) Fluidic element (b) System graph

Figure 2.19: System graph of a gluidic line with small diameter.

Another type of element in the lumped fluidic systems is a small fluidic line with comparatively large cross sectional area. Since the length of a line is assumed to very small compared to the diameter of the line, $P_1 = P_2$. Further from the matrix equation of this element (equation (2.36)),we obtain

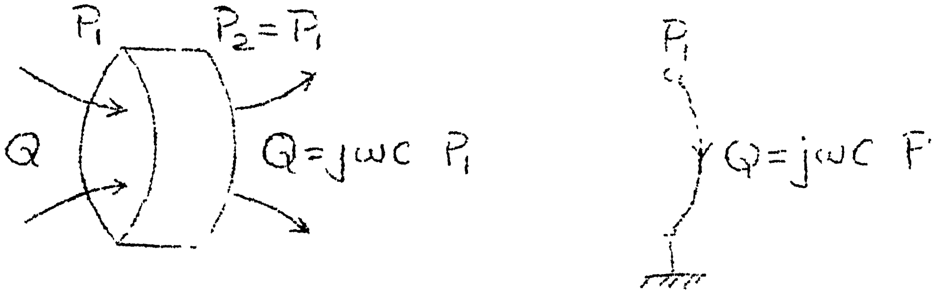
$$\begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ j\omega C & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ Q_2 \end{bmatrix}$$

If we denote net flow by $Q = Q_1 - Q_2$ then

$$Q = j\omega C$$

where $C = \frac{A}{\rho a^2} \ell$

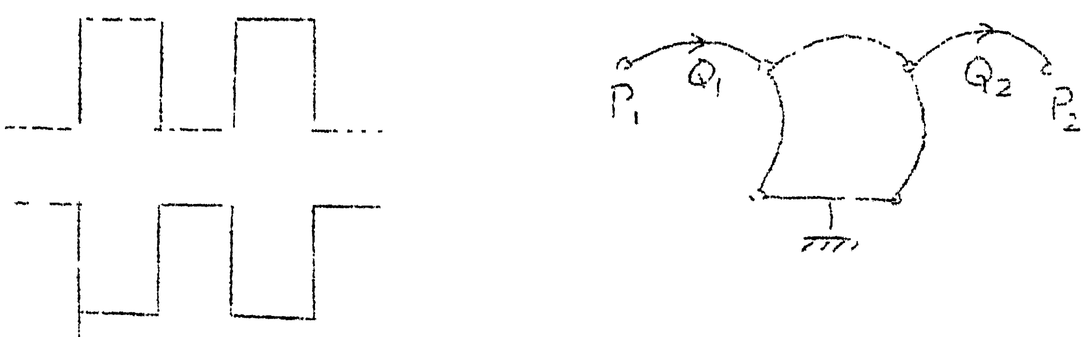
The system graph reduces to two terminal representation with one terminal grounded as shown in Figure 2.20.



(a) Fluidic element (b) System graph

Figure 2.20: System graph representation of a fluidic line with large diameter.

Thus we see that the system graph representation confirms the electrical analogy we have developed in Section 2.4. Having found the way of realising inductance and capacitance with fluidic structures, we can easily convert an electrical filter design into a fluidic filter. This can be appreciated by examining the system graph of composite fluidic system with inductive and capacitive fluidic elements connected in cascade as shown in Figure 2.21.



(a) Fluidic system (b) System graph

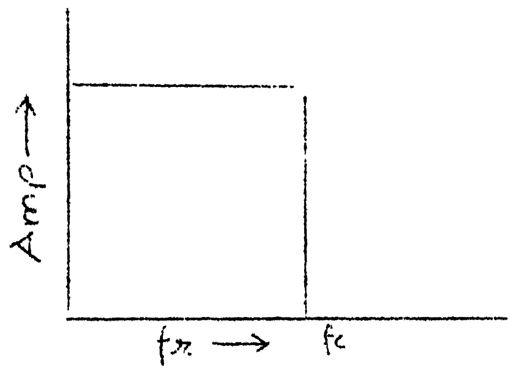
Figure 2.21: System graph of a lumped parameter fluidic system.

The system graph of the combined system of Figure 2.21 has been obtained by connecting the system graph of individual elements in series. The structure so obtained resembles the ladder network of electrical system. The standard electrical filters exist in this form of ladder network. We can, therefore, easily convert these filter networks into fluidic design using analogy established in Section 2.4.

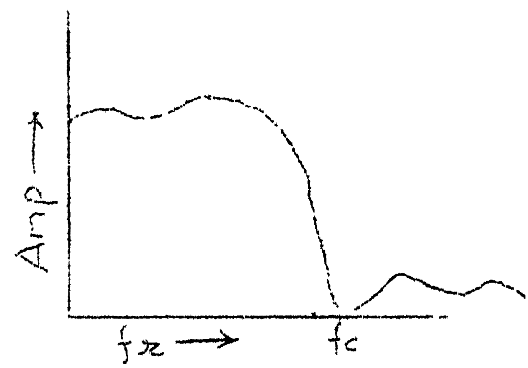
SECTION 3

ELECTRICAL FILTER THEORY

A filter is a network which passes a desired band of frequencies but suppresses the other frequency components. Ideally, a filter should pass all the frequencies in the desired band without any attenuation, whereas it should completely suppress the other frequencies. In other words the characteristic of an ideal filter has got a square shape with sharp cut off as shown in Figure 3.1a. Such an ideal performance is not practically possible but can only be approximated to have a form such as the one shown in Fig. 3.1b.(3)



(a) Characteristic of an ideal filter



(b) Characteristic of a practical filter

Figure 3.1: Ideal and practical characteristics of a filter.

As shown in Fig. 3.1b, a practical filter has a gradual roll off. The steepness of characteristic can be increased by increasing the number of elements. The simplest

example of a filter network is a resonance circuit consisting of a series or parallel combination of an inductance and capacitance. It passes the frequencies near resonance and suppresses the other frequencies. An actual filter, however, is more complicated in structure than a resonant circuit and typically consists of a ladder network with suitable combination of inductances and capacitances.

The work on filter started in the 1920s with the introduction of image parameter concept. The early techniques based on image parameter theory gave way to what is known as modern filter theory. For elaborate filter specifications it is the modern filter theory which is preferred. We have discussed both theories in this section.

Modern filter theory was originally developed in relation to lumped network elements e.g. inductances and capacitances. However, the theory was later extended to microwave filters in which basic elements were made out of transmission line sections. The microwave filter structure bear a close analogy to fluidic system elements. We have therefore, discussed some essential features of microwave filters with a view to adopt them for fluidic filters.

3.1 IMAGE PARAMETER FILTERS:

We will consider the symmetrical T network of Figure 3.2 to explain the image parameter concept. The input

impedance of this network is given by equation (3.1).

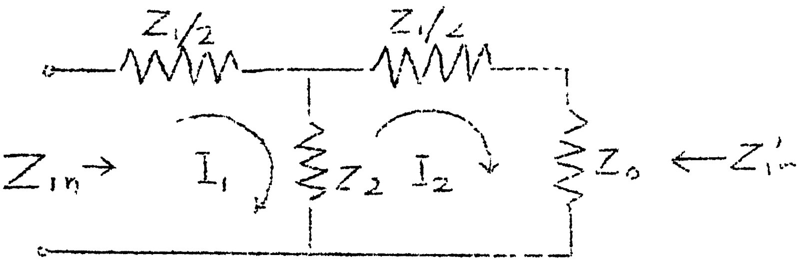


Figure 3.2: Symmetrical T section

$$Z_{in} = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_o\right)}{\frac{Z_1}{2} + Z_2 + Z_o} \qquad (3.1)$$

If $Z_o = Z_{in}$ then equation (3.1) gives

$$Z_o = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2} \right)} \qquad (3.2)$$

A practical ladder network consists of a series of such identical T sections finally terminated in Z_o. The input impedance of such a ladder network is equal to Z_o. This condition is met from both ends of the ladder. Filter circuits using such networks are called image parameter filters.

Now to examine the filtering behaviour of this network let us denote the current ratio $\frac{I_1}{I_2}$ by e^{Γ}. Then

$$\frac{I_1}{I_2} = \frac{Z_1/2 + Z_2 + Z_o}{Z_2} = e^{\Gamma} \qquad (3.3)$$

$$\cosh \Gamma = \frac{e^{\Gamma} - e^{-\Gamma}}{2} = 1 + \frac{Z_1}{2Z_2} \qquad (3.4)$$

or

$$\sinh \frac{\Gamma}{2} = \sqrt{\frac{1}{2}\left(\cosh \frac{\Gamma}{2} - 1\right)}$$

$$= \sqrt{\frac{Z_1}{4Z_2}} \tag{3.5}$$

Γ is denoted by $\alpha+j\beta$ where α is the attenuation constant and β is the phase constant. Then $\sinh \frac{\Gamma}{2}$ can be expressed as given by equation (3.6).

$$\sinh \frac{\Gamma}{2} = \sinh \frac{\alpha}{2} \cosh \frac{\beta}{2} + j \cosh \frac{\alpha}{2} \sinh \frac{\beta}{2} \tag{3.6}$$

A simple filter network consists of series and shunt arms of opposite reactances. The term $\frac{Z_1}{4Z_2}$ will, therefore, be negative. This means equation (3.6) will have only imaginary value; its real value will be zero i.e.

$$\sinh \frac{\alpha}{2} \cos \frac{\beta}{2} = 0 \tag{3.7}$$

$$\text{and } \cosh \frac{\alpha}{2} \sin \frac{\beta}{2} = \sqrt{\frac{Z_1}{4Z_2}} \tag{3.8}$$

A filter network is characterised by the pass band and stop band. The passband is the frequency region where most of the signals are passed without any significant attenuation. The remaining region of the frequency axis where the signals undergo certain attenuation, is known as stop band. The frequency at which the network changes its behaviour from pass band to stop band is known as cut off frequency.

For the symmetrical T network that we have considered, the pass band will be given by the region where $\alpha = o$, whereas the stop band will be given by the region where $\alpha \neq o$. From the equations(3.7) and (3.8), these two conditions can be interpreted as given below.

a) for $\alpha = o$

$$\beta \neq o$$

$$\sinh\frac{\beta}{2} = \sqrt{Z_1/4Z_2}$$

The minimum value $\sinh\frac{\beta}{2}$ can have is zero. Therefore, we can say that the pass band starts from a frequency where $\frac{Z_1}{4Z_2} = o$.

b) for $\alpha \neq o$

$$\beta = o$$

$$\text{or,}\quad \cosh\frac{\alpha}{2} = \sqrt{Z_1/4Z_2}$$

The minimum value $\cosh\frac{\alpha}{2}$ can have is 1. Therefore we can say that the stop band starts from $\frac{Z_1}{4Z_2} = -1$.

The pass band therefore will be given by

$$0 < \left|\frac{Z_1}{4Z_2}\right| < 1$$

and the cut off frequency is given by

$$\left|\frac{Z_1}{4Z_2}\right| = 1$$

Thus, we see that we can obtain the filtering effect by using a symmetrical ladder network in which the series and shunt arms are of opposite reactance value.

3.1.1 Constant K Low Pass Filter:

In the ladder network we considered above, Z_1 and Z_2 are of opposite nature in reactance. We can therefore put $Z_1 Z_2 = K^2$ where K is a constant independent of frequency. Such networks are called constant K filters [3]. As an special case let us consider the network of Figure 3.3 with $Z_1 = j\omega L$ and $Z_2 = -j/\omega C$.

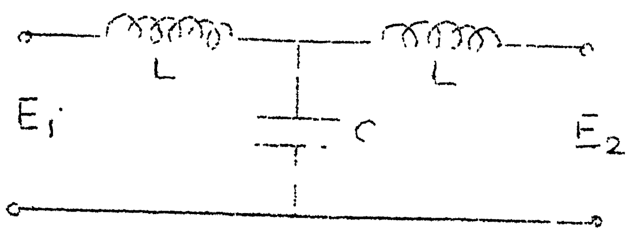


Figure 3.3: Constant K low pass filter

The cut off frequency will be given by

$$Z_1 = -4Z_2$$

or $$j\omega_c L = \frac{4j}{\omega_c C}$$

or, $$f_c = \frac{1}{\pi\sqrt{LC}} \qquad (3.9)$$

The pass band will extend from

$Z_1 = o$

or $\omega L = o$

or $\omega = o$

The frequency characteristic of constant K filter is given in Figure 3.4, which is that of a low pass filter. One section of a constant K filter gives a roll off in pass band of 6 dB per octave. If a higher roll off is required number of such sections can be connected in cascade.

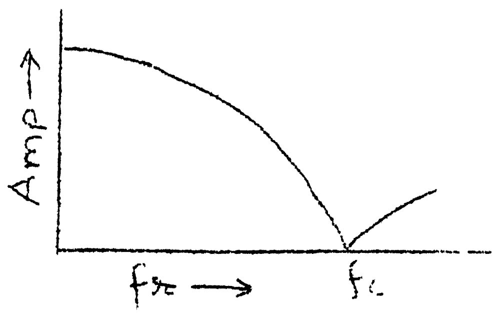


Figure 3.4: Frequency characteristic of constant K filter.

An inherent drawback with the constant K filter is that its image impedance is constantly changing with frequency in a manner that does not correspond to any lumped element. This can be seen from the equation (3.10).

$$Z_o = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)} \tag{3.10a}$$

$$= \sqrt{\frac{L}{C}\left[1-\left(\frac{f}{f_c}\right)^2\right]} \tag{3.10b}$$

For any lumped element termination, the filter characteristic will deviate from the normal since the characteristic impedance is not exactly realised. To see that the characteristic impedance is exactly obtained at least at one frequency e.g. $\omega = o$, the filter is normally terminated in a zero frequency load i.e.

$$R = \sqrt{\frac{L}{C}}$$

3.1.2 The m-derived Filter:

We can obtain a steeper roll off characteristic if we replace the shunt capacitance of a constant K filter by a series resonance circuit, as given in Figure 3.5. This network is known as m-derived filter. The shunt arm acts as virtual short at the resonance frequency giving a very high attenuation. This frequency is denoted by f_∞.

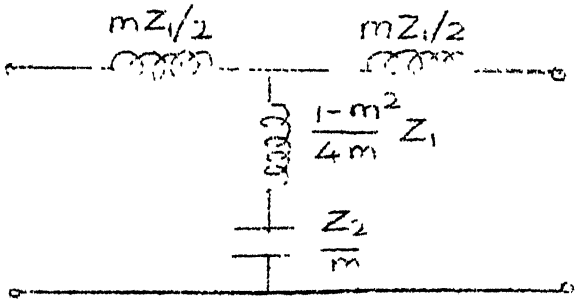


Figure 3.5: The m-derived low pass filter

The filter can be derived from constant K filter by putting $Z_1' = mZ_1$. The value of Z_2' can be then, found out by equating $Z_0' = Z_0$,

or,

$$\frac{(mZ_1)^2}{\ } + m\, Z_1 Z_2' = \frac{Z_1{}^2}{4} + Z_1 Z_2$$

or,

$$Z_2' = \frac{Z_2}{m} + \frac{1-m^2}{4m}\, Z_1 \qquad (3.11)$$

The resonance frequency is therefore, given by

$$\frac{Z_2}{m} = \frac{1-m^2}{4m}\, Z_1$$

or,

$$\frac{1}{2\pi f_\infty mc} = \frac{1-m^2}{4m}\, 2\pi f_\infty L$$

or,

$$f_\infty = \frac{1}{\pi\sqrt{(1-m^2)LC}} \qquad (3.12)$$

By putting $\sqrt{LC} = \frac{1}{\pi f_c}$ equation (3.12) gives

$$m = \sqrt{1-(f_\infty/f_c)^2} \qquad (3.13)$$

Apart from sharp cut off the m-derived section has got another major advantage that fairly a constant image impedance can be obtained by proper selection of m. To demonstrate this let us break the m derived section into 2 half L sections as shown in Figure 3.6. For this network

$$Z_{2i} = \sqrt{\left(\frac{mZ_1}{2} + \frac{1-m^2}{2m} Z_1 + \frac{2Z_2}{m}\right) \frac{mZ_1}{2}}$$

$$= \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$$

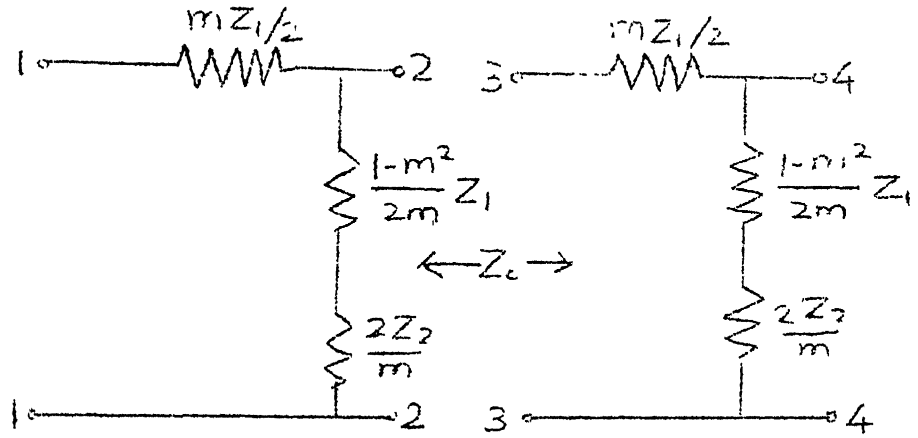


Figure 3.6: m-derived filter devided in to two L sections

Thus the image impedance of L section is same as that of constant K filter section. Also, Z_{1i} can be derived as

$$Z_{1i} = \frac{R_K[1-(1-m^2)\ f^2/f_c^{\ 2}\ ]}{\sqrt{(1-f^2/f_c^{\ 2}\ )}} \tag{3.14}$$

It can be seen that if we used m = 0.6; a nearly constant value of image impedance can be obtained which is equal to R_K. Hence we see that m derived half sections can be conveniently used for proper termination.

A major disadvantage of m derived filter is that it gives smaller attenuation in stop band. For this reason, a practical filter normally consists of a number of sections of constant K and m derived type suitably connected to form a ladder

network. We will illustrate the practical filter design by taking an example.

Example 3.1:

Let us design a low pass filter with a cut off frequency of 1000 Hz with a very high attenuation at 1250 Hz and ∞. The filter is to be terminated in 500 Ω.

For getting a very high attenuation at ∞, we use a constant K section with

$$L = \frac{R}{\pi f_c} = \frac{500}{\pi \times 1000} = 0.159 \text{ H}$$

so that L/2 is 0.079 H

$$C = \frac{1}{\pi f_c R} = \frac{1}{\pi \times 1000 \times 500} = 0.636 \ \mu\text{F}$$

The m-derived section with f_∞ = 1250 Hz will have the value of m as given below.

$$m = \sqrt{1 - (f_c/f_\infty)^2} = \sqrt{1 - (1000/1250)^2}$$
$$= 0.6$$

This m derived section will be most suitable for termination bynbreaking it into 2 halves since for this m = 0.6.

$$\frac{mL}{2} = \frac{0.6 \times 0.159}{2} = 0.0477\text{H}$$

$$\frac{1-m^2}{2m} L = \frac{1-0.36}{1.2} \times 0.159 = 0.0848 \text{ H}$$

$$\frac{mc}{2} = \frac{0.6 \times 0.636}{2} = 0.191 \ \mu\text{F}$$

The composite filter is given in Figure 3.7.

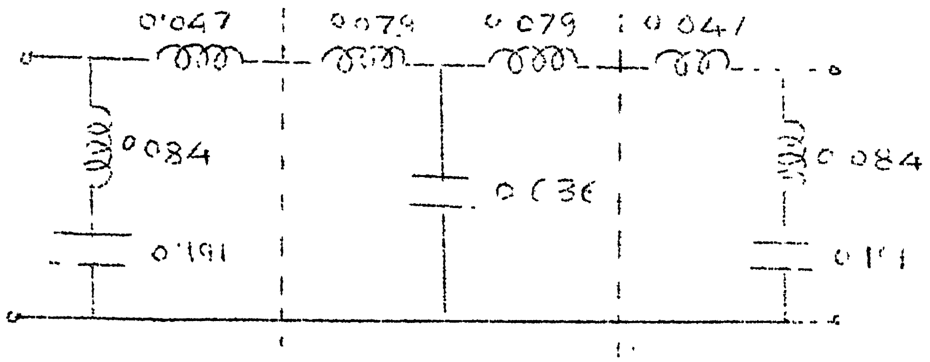


Figure 3.7: Composite filter design of example 3.1

The elements can be combined to form the final filter design as shown in Fig. 3.8.

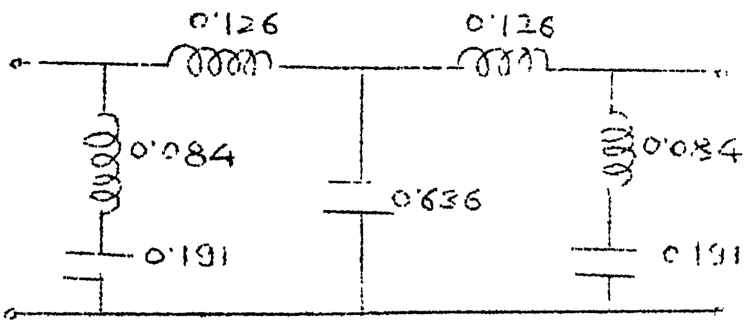


Figure 3.8: Final filter design of example 3.1

3.2 MODERN FILTER THEORY:

The image parameter theory is based on analysis of certain network structures. In contrast, the modern filter theory relies on proper synthesis in which one starts with the specifications and arrive at a network structure that

meets requirements exactly. The specifications include even the tolerances for the approximations. There are families of certain standard transfer functions which are used for filter approximation and the ready made tables of element values to suit these transfer functions [9]. To illustrate the synthesis procedure from the first principle we will take an example of a simple transfer function in the following example.

Example 3.2:

Let us consider a transfer function

$$T(s)=\frac{1}{S^3+2S^2+2S+1} \qquad (3.15)$$

The response of this filter can be found by putting different values of ω in the above expression. (Figure 3.9). It is basically a low pass filter.

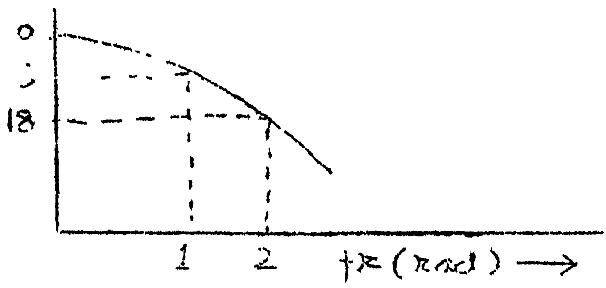


Figure 3.9: Frequency response of a filter

If we solve the denominator we get 3 poles i.e.

-1

-0.5 ± 0.866

$$Z_{11} = \cfrac{1}{S + \cfrac{1}{\cfrac{2S^2+2S+1}{S+1}}}$$

or,

$$Z_{11} = \cfrac{1}{S + \cfrac{1}{\cfrac{2S+1}{S+1}}} \tag{3.17}$$

Now we can realise this driving point function with the mere examination of equation (3.17). The filter configuration will therefore appear as given in Figure 3.11.

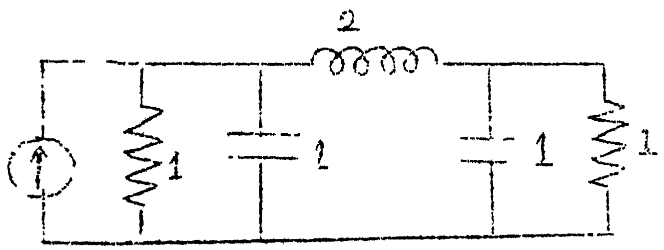


Figure 3.11: Low pass filter of example 3.2

This filter has got a 3 dB cut off frequency of 1 rad/sec and it terminates in a resistence of 1 Ω. We can convert this design to appropriate cut off frequency f_c and load R by following relations.

$$L' = \frac{L \times R}{2\pi f_c} \tag{3.18a}$$

$$C' = \frac{C}{2\pi f_c \times R} \tag{3.18b}$$

This point can be verified by seeing the effect of the change in element values on the transfer function and the frequency response characteristic. The frequency response characteristic will get normalised to the new cut off frequency; the shape of the curve will remain same. Similarly the element values get normalised so as to suitably match the new value of load impedance.

From the above example, it is clear that we can design a filter with any cut off frequency and load from the elementary design of Figure 3.11. The frequency response characteristic will have the same shape when normalised to a cut off frequency of 1 rad/sec. This concept has led to simplification of procedure involved in filter design with the help of standard tables which are worked out before hand.

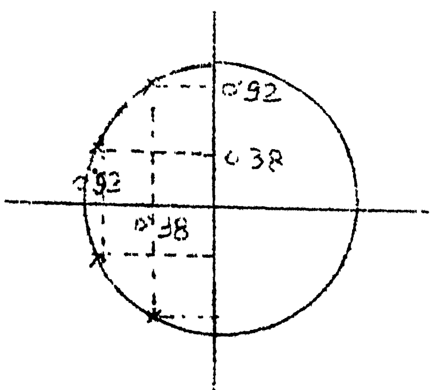
In the example 3.2 we have considered the values of load and source impedance as same. Similar standard designs can be obtained for different ratios of source to load impedance. Example 3.2 illustrates the design of a 3rd order filter. The filter has got a very gradual roll off characteristics. We can obtain a steeper characteristic by increasing the order of the filter. However, if we want that the basic nature of the frequency response curve should not change, then the new transfer function should also belong to the same family of curves.

The transfer function of example 3.2 is identified by the fact that all its poles lie on the circumference of unit circle. Such family of curves are known as Butterworth characteristic. The method of approximating the ideal filter response with the help of this family of curves can be said to be Butterworth approximation. Similarly the denominator of the transfer function will be known as Butterworth polynomial.

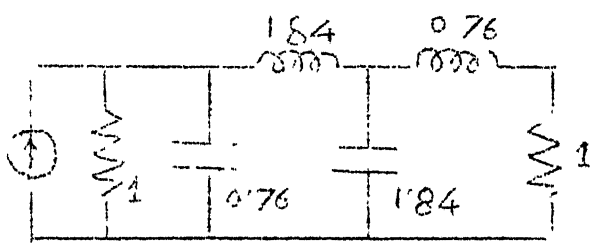
Working on the same lines, a fourth order Butterworth polynomial will be

$$S^4+2.61S^3+3.41S^2+2.61S+1$$

The location of poles for this transfer function and the filter configuration obtained are given in Figure 3.12.



(a) Pole location



(b) Filter configuration

Figure 3.12: Design of fourth order Butterworth filter

So far we have used the circuits with current generator. The same design can be used for a circuit with voltage source by slight rearrangements of element values as shown in

Figure3.13. The figure shows the circuit for 4th order Butterworth filter.

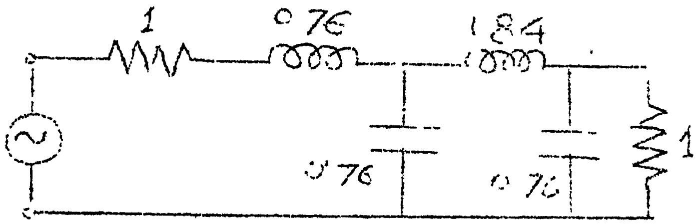


Figure 3.13: Fourth order Butterworth filter for circuit with voltage source.

Working on these lines we can make a table of element values for different values of n (order of filter) and R_L/R_S (ratio of load to source impedance). Such a table for Butterworth family of curves is given in Appendix 'C'. The attenuation characteristics are given in Appendix 'B'. With these two things given, the job of filter design becomes easy. All that one has to do can be given in following steps:

i) See the filter specifications

ii) Select a curve which fits in best into the specifications.

iii) For that order of filter and given ratio R_L/R_S, select the element values from the table.

iv) Normalise the design for given cut off frequency and load impedance.

Although Butterworth filters are most commonly used because of their simplicity, there are other filter characteristics which have other attractions like more steepness,

less ripple, uniform delay etc. In the following paragraphs, we will give a brief introduction to the different kinds of filter approximations [10].

i) Butterworth filters:

The Butterworth approximation to an ideal low pass filter is based on the assumption that a flat response at zero frequency is more important than sharp cut off. The normalised transfer function has roots which fall on a unit circle.

ii) Chebychev response:

A sharp cut off can be obtained if we move the poles of Butterworth filter to right by multiplying them by a constant. The poles will now lie on an ellipse instead of unit circle. The sharpness is however obtained at the expense of ripple in the pass band. The ripples will grow in magnitude as the real part of the poles is decreased. A table of Chebychev filter will specify the magnitude of ripple for which it is applicable. This type of characteristic is used when a sharp cut off is required with less number of elements.

iii) Bessel filters:

We have seen that Chebychev filters give better selectivity but only at the expense of poor transient behaviour. The Bessel transfer function has been optimised to obtain a linear phase i.e. maximally flat delay. Bessel

response, essentially has no ripples but it is less selective then the other types of filters. The poles of Bessel function also lie on unit circle but with their imaginary parts separated by an amount given by 2/n. The Bessel filters are used in applications where the transient properties are of major importance.

iv) Elliptic-function filters:

All the filters discussed above are of all pole type i.e. they have a denominator polynomial. They can get infinite attenuation only at $f \rightarrow \infty$. Elliptic function filters have zeros as well as poles in the transfer function. The zeros at finite frequencies reduce the transition region so much that extremely sharp roll off characteristic can be obtained. The improved performance is obtained at the expense of return lobes in the stop band.

The elliptic filters, although, give superior performance, are more complex in design. A low pass elliptic filter uses a series resonance circuit in place of shunt capacitance as given in Figure 3.14.

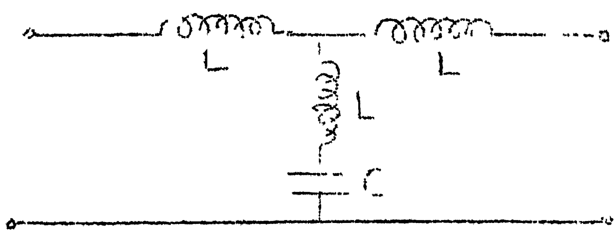


Figure 3.14: A low pass elliptic filter.

We will now consider an example to illustrate how a required filter can be designed using the standard tables.

Example 3.3:

Let us design a low pass filter with 3 dB attenuation a 1000 Hz.

A minimum 20 dB attenuation at 2000 Hz is required

$R_S = R_L = 500\ \Omega$

The normalised requirement says that we want 20 dB attenuation at $\frac{2000}{1000} = 2$ rad/sec. Examination of attenuation curve given in Appendix 'B' indicates that an n=4 Butterworth filter will satisfy the requirements.

The element values for fourth order Butterworth filter can be obtained from Appendix 'C'.

$$L_1 = 0.765$$
$$C_2 = 1.848$$
$$L_3 = 1.848$$
$$C_4 = 0.765$$

These values can be normalised for given cut off frequency (i.e. 1000 Hz) and load by using relations

$$L' = \frac{LR}{2\pi f_c}$$

$$C' = \frac{C}{2\pi f_c \times R}$$

$$L'_1 = \frac{0.765 \times 500}{2\pi \times 1000} = 0.0564 \text{ H}$$

$$C'_2 = \frac{1.848}{2\pi \times 1000 \times 500} = 0.598 \ \mu\text{F}$$

$$L'_3 = \frac{1.848 \times 500}{2\pi \times 1000} = 0.136 \text{ H}$$

$$C'_4 = \frac{0.765}{2\pi \times 1000 \times 500} = 0.243 \ \mu\text{F}$$

The resulting filter is shown in Figure 3.15.

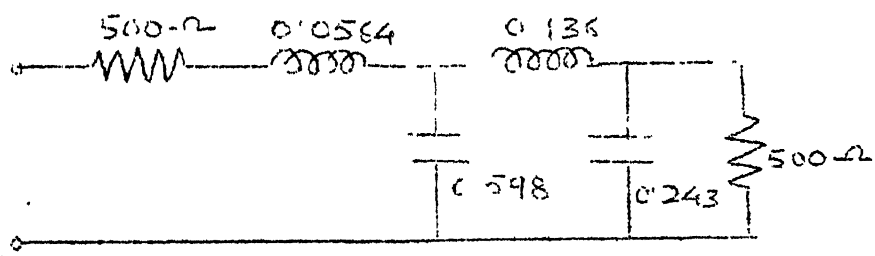


Figure 3.15: The resultant filter design of example 3.3

3.3 ESSENTIAL FEATURES OF MICROWAVE FILTERS:

In a normal electrical circuit which operates at moderate frequencies, the inductances and capacitances are realised through wire wound coils and condensors. At higher frequencies, such as those in the microwave range, the dimensions of coils and condensors may become comparable to the wavelength. Then, we cannot think of an inductance and capacitance in isolation without considering the effect of

wave propagation. This necessity has led to the concept of using transmission line networks for high frequency filters.

The filter theory discussed in this section can be used for any transmission line network. However we present it under the heading of microwave filters since it has been extensively used and developed in the field of microwaves.

3.3.1 Richard's Transformation:

Richard's suggested that transmission lines of commensurate lengths can be used as inductance and capacitance through suitable frequency transformation [11]. To appreciate this fact let us examine the equation (2.8) which gives the input impedance of a transmission line terminated in a load impedance of Z_L.

$$Z_{in} = \frac{Z_L \cosh \Gamma l + Z_c \sinh \Gamma l}{Z_L \sinh \Gamma l + Z_c \cosh \Gamma l}$$

The input impedance of the lines which are short circuit or open circuit at far end can be found out from this as

$$Z_{sh} = Z_c \tanh \Gamma l \tag{3.19a}$$

$$Z_{op} = Z_c / \tanh \Gamma l \tag{3.19b}$$

If we replace $\tanh \Gamma l$ by λ then equations (3.19) becomes

$$Z_{sh} = Z_c \lambda \tag{3.20a}$$

$$Z_{op} = Z_c / \lambda \tag{3.20b}$$

This shows that if one considers λ as an independent frequency variable, Z_{sh} is the impedance of an inductance $L = Z_c$ and Z_{op} is that of a capacitance $C = 1/Z_c$. The frequency transformation $\lambda = \tanh \Gamma \ell$ is known as Richard's transformation [12].

The elements L and C can be realised through transmission lines by using Richard's transformation as given in Figure 3.16. Resistence is independent of frequency and, therefore, is invariant in the above transformation.

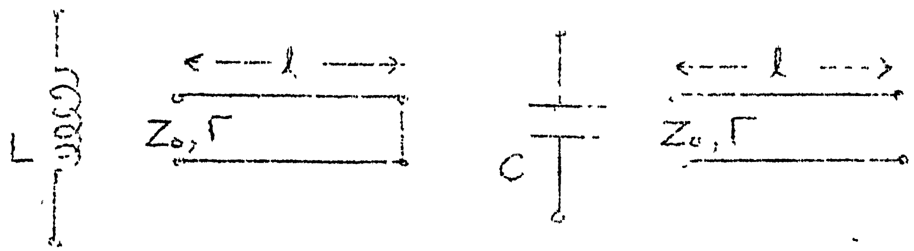


Figure 3.16: Realisation of inductance and capacitance using transmission lines.

For a lossless line, equation $\lambda = \tanh \Gamma \ell$ can be further simplified

$$\lambda = \tanh j\beta \ell$$

$$\text{or } \lambda = \tan \beta \ell$$

But $\beta = \frac{\omega}{a}$ where a is the speed of wave propagation

$$\therefore \quad \lambda = \tan \frac{\omega}{a} \ell$$

The factor $\frac{\ell}{a}$ signifies a constant delay, T, depending upon length of transmission line, ℓ. The length ℓ has to be

maintained during the transformation. Therefore all the elements of transmission line network will have same length. This is a fundamental condition we put on a transmission line networks for ease of design. When such is the case, each transmission line section is said to be of unit length.

The salient features of $S \rightarrow \lambda$ mapping are shown in Figure 3.17. The frequency domain bounded by two straight lines $S = \sigma \pm \frac{\pi}{2T}$ carries all the information of the λ plane. The entire real axis of S plane is transformed into segment $|\lambda| < 1$.

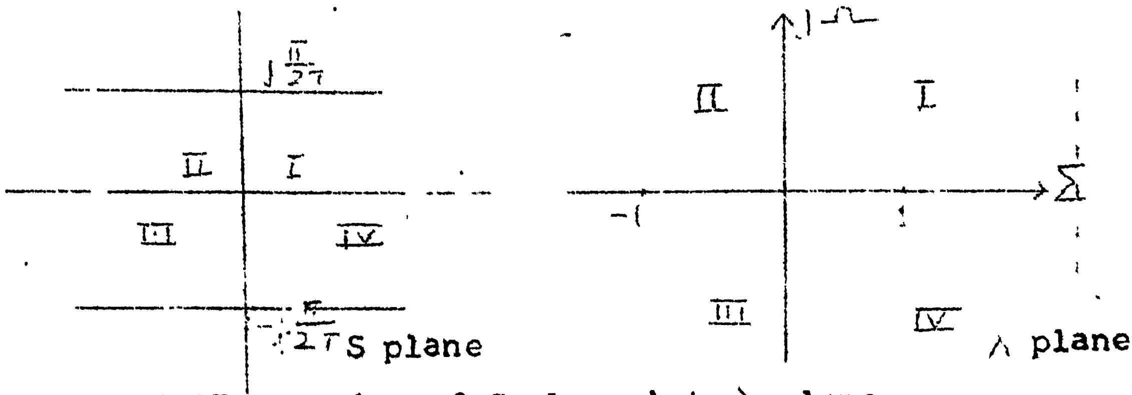


Figure 3.17: Mapping of S plane into λ plane.

Since λ is periodic function of ω the properties of Richard's network are, also periodic as shown in Figure 3.18. The periodicity of λ is given by $\frac{\omega}{a} \ell = \frac{\pi}{2}$. A low pass design in λ -domain will appear as a comb filter in ω domain i.e., one with a periodic pass and stop bands as shown in Figure 3.19.[13].

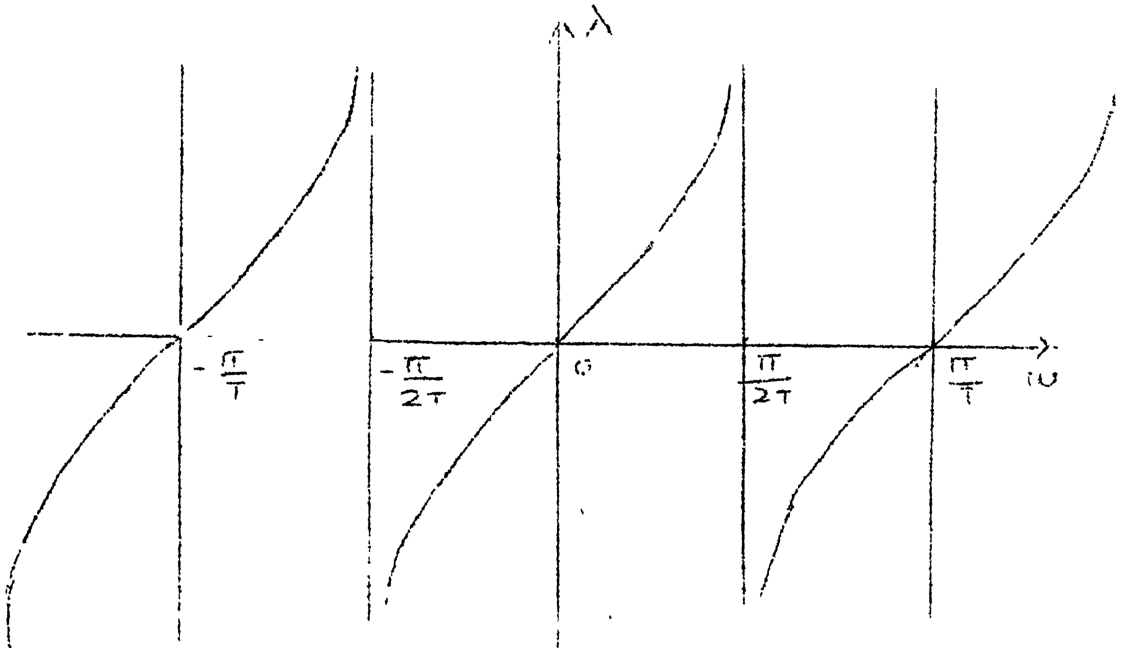


Figure 3.18: Periodicity of λ with respect to ω

The pass bands of real frequency axis will appear with a periodicity of $\frac{\pi}{T}$. The low pass effect is obtained by proper selection of delay T such that the frequency band of interest falls before the rising edge of second pass band. The delay T in turn depends on the unit length ℓ.

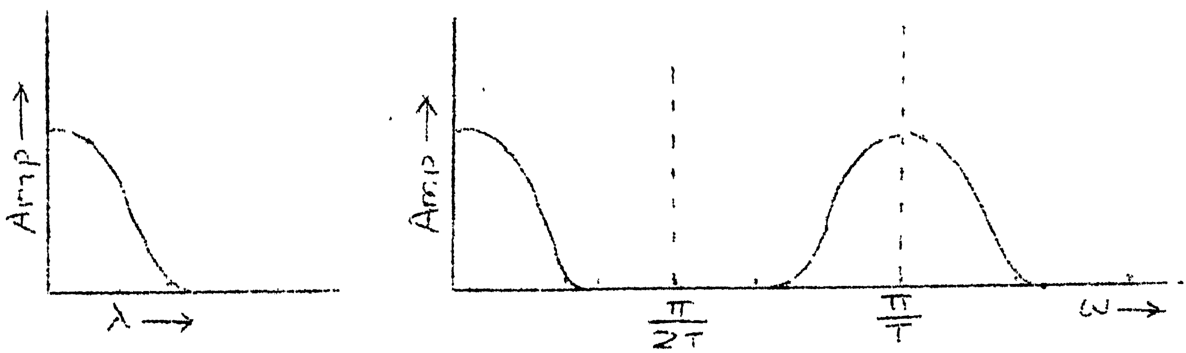


Figure 3.19: Transformation of a low pass design from to ω frequency axis.

Having studied the behaviour of $S \rightarrow \lambda$ transformation, let us see how can we combine the short and open circuit lines to obtain a filter structure. These configurations can only be used as stub lines in a transmission line network, since the loading at the far end is prespecified. These stubs are interconnected by means of what is known as a unit element. A unit element is a lossless transmission line of unit length. The transfer matrix of the UE obtained from the equation (2.7) is given below.

$$\begin{bmatrix} \cosh \Gamma l & Z_c \sinh \Gamma l \\ \frac{1}{Z_c} \cdot \sinh \Gamma l & \cosh \Gamma l \end{bmatrix}$$

$$= \cosh \Gamma l \begin{bmatrix} 1 & Z_c \tanh \Gamma l \\ \frac{1}{Z_c} \tanh \Gamma l & 1 \end{bmatrix}$$

Using transformation $\lambda = \tanh \Gamma l$, the transfer matrix of UE becomes

$$(1-\lambda^2)^{\frac{1}{2}} \begin{bmatrix} 1 & \lambda Z_o \\ \lambda / Z_o & 1 \end{bmatrix} \qquad (3.21)$$

3.3.2 <u>Kuroda's Identity:</u>

Designing filters with the help of transmission line

elements is not the same as it is with lumped elements. One cannot obtain all sorts of series, parallel combinations with transmission line networks because of their peculiarities of construction. In a coaxial cable, for example, the inductance or capacitance will exist between the central conductor and outer sheath. Therefore the series connection in this case is not feasible.

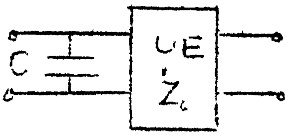
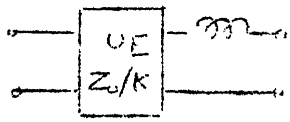
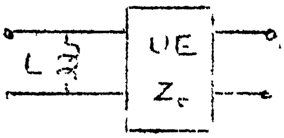
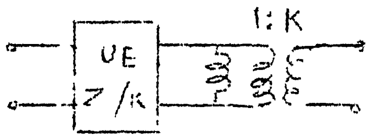
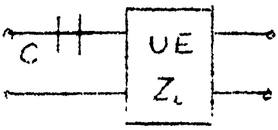
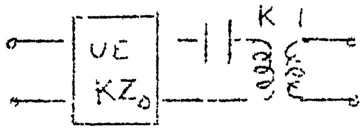
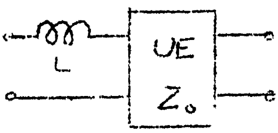
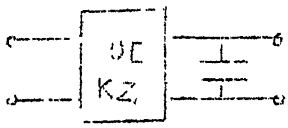
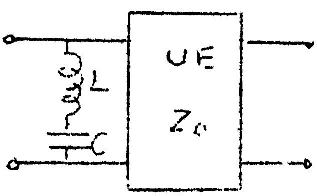
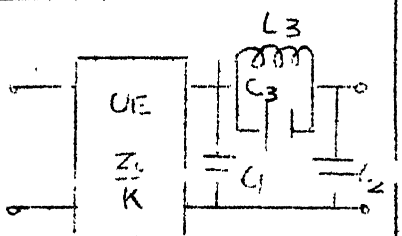
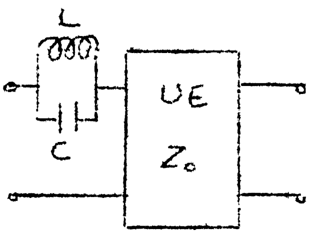
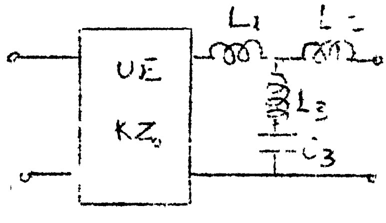
We are thus constrained to use only those configurations in which both inductive and capacitive elements appear in parallel. Since in a standard low pass design, we need to have the effect of an inductance in series and not in shunt, this poses realisation difficulties.

To overcome these difficulties, Kuroda proposed a list of identities as given in Table 3.1. [12]. The use of these identities lie in the fact that by combining with UEs, an unrealisable filter network can be converted to a realisable network by applying these identities repeatedly. A simple example is that of a hypothetical series inductance which when combined with a UE, can be replaced by an equivalent combination of a UE with a shunt capacitance which can be realised in practice.

Let us consider an identity from the table and try to verify it as an exercise [14]. We will consider identity

Table 3.1

List of Kuroda's Identities

Sl. No.	Original Circuit	Equivalent Circuit	Relation
1			$K = 1 + Z_0 C$ $L = \frac{K-1}{K} Z_0$
2			$K = 1 + \frac{Z_0}{L}$ $L = \frac{Z_0}{K(K-1)}$
3			$K = 1 + \frac{1}{Z_0 C}$ $C = \frac{1}{Z_0} \frac{1}{K(K-1)}$
4			$K = 1 + \frac{L}{Z_0}$ $C = \frac{1}{Z_0} \frac{K-1}{K}$
5			$K = 1 + \frac{Z_0 C}{1+LC}$ $C_1 = \frac{KLC}{Z_0}$ $C_2 = -\frac{LC}{Z_0}$ $L_3 = \frac{K-1}{K} Z_0$ $C_3 = \frac{KLC}{(K-1)Z_0}$
6			$K = 1 + \frac{L/Z_0}{1+LC}$ $L_1 = KLCZ_0$ $L_2 = -L_1$ $L_3 = \frac{KL}{K-1} C Z_0$ $C_3 = \frac{K-1}{KZ_0}$

1 for this purpose. The matrix representation of original circuit will be

$$\frac{1}{\sqrt{(1-\lambda^2)}} \begin{bmatrix} 1 & 0 \\ \lambda c & 1 \end{bmatrix} \begin{bmatrix} 1 & \lambda Z_o \\ \frac{\lambda}{Z_o} & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{(1-\lambda^2)}} \begin{bmatrix} 1 & \lambda Z_o \\ (C+\frac{1}{Z_o})\lambda & Z_o C \lambda^2 + 1 \end{bmatrix} \qquad (3.22)$$

Similarily the matrix representation of equivalent circuit is

$$\frac{1}{\sqrt{(1-\lambda^2)}} \begin{bmatrix} 1 & \lambda Z_o' \\ \frac{\lambda}{Z_o'} & 1 \end{bmatrix} \begin{bmatrix} 1 & \lambda L \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{(1-\lambda^2)}} \begin{bmatrix} 1 & (Z_o'+L)\lambda \\ \frac{\lambda}{Z_o'} & \frac{\lambda^2 L}{Z_o'} + 1 \end{bmatrix} \qquad (3.23)$$

For the two networks to be equivalent all the elements of matrices (3.22) and (3.23) should be equal.

$$\therefore \quad Z_o = Z'_o + L \tag{3.24a}$$

$$C + \frac{1}{Z_o} = \frac{1}{Z'_o} \tag{3.24b}$$

$$\frac{L}{Z'_o} = Z_o C \tag{3.24c}$$

If we define K by $1+Z_oC$ as in table, then

$$Z'_o = \frac{Z_o}{K} \tag{3.25a}$$

$$L = \frac{K-1}{K} Z_o \tag{3.25b}$$

This is same as the relations given in the table. All the identities can be varified on the same lines. Later on in Section 5 we will use some of these identities for the synthesis of fluidic filters.

3.3.3 Important Theorems:

In this section we will present two important theorems connected with Richards' transformation which serve as the basis of obtaining microwave filter structure using unit elements. These theorems will be used later for fluidic filter design [12].

Theorem 1: Richards' Theorem

It is always possible to realise a positive real function $Z(\lambda)$ with a unit element cascaded with another

positive real function $Z_1(\lambda)$ with a degree not higher than $Z(\lambda)$. Refering to Figure 3.20, $Z(\lambda)$ can be written as

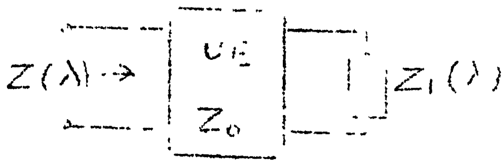


Figure 3.20: Simplification of Richards' networks

$$Z(\lambda) = \frac{Z_o[\lambda Z_o + Z_1(\lambda)]}{[Z_o + Z_1(\lambda)]} \qquad (3.26)$$

equation (3.26) can be rearranged to find out $Z_1(\lambda)$ which will satisfy this condition.

$$Z_1(\lambda) = \frac{Z_o[Z(\lambda) - Z_o]}{[Z_o - Z(\lambda)]} \qquad (3.27)$$

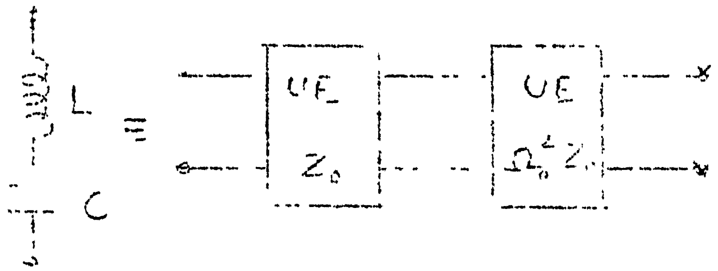
Characteristic impedance of UE, Z_o can be found out by putting $\lambda = 1$ in equation (3.27) so that

$$Z_o = Z_o(1)$$

Richards' theorem does not guarantee the realisation of a transfer function with the help of minimum number of elements. But it is important because of the generality with which it can be applied to the transmission line network. It can be resorted to when the other standard methods do not give the desired result.

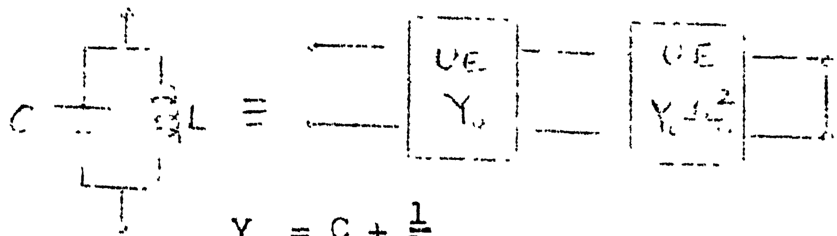
Theorem 2:

A reactance of degree n can be realised as input immittance of n cascaded UE_s short or open circuited at the far end. Some examples of the circuits of this nature are shown in Figure 3.18.



$$Z_o = L + \frac{1}{C}$$

(a) Series resonant circuit



$$Y_o = C + \frac{1}{L}$$

(b) Shunt resonant circuit

Fig. 3.21: The bar-type equivalents of LC resonators ($\lambda_o^2 = 1/LC$).

In the above paragraphs we discussed some essential aspects of electrical filter theory which are subsequently used for understanding and designing of fluidic filters.

In the field of fluidic filters image parameter technique has been widely used so far. However, the concepts of modern filter theory does not seem to be extended to this area. In later sections we have laid emphasis on the use of modern filter theory which leads to a more systematic approach than the image parameter technique. We discussed some essential aspects of microwave filters since the microwave filter structures bear a strong resemblence to fluidic systems and the design procedure they suggest for fluidic system.

SECTION 4

INTRODUCTION TO FLUIDIC FILTER DESIGN

In this section we have discussed the fluidic filter design and analysis using conventional techniques. It has been broadly categorised in two classes: i) Lumped parameter filters ii) distributed parameter filters. The first category of filters uses the lumped 2-terminal elements for realisation. On the other hand the later one uses sections of transmission line in cascade and as stubs.

In the fluidics, the lumped parameter filters are so far mostly designed using image parameter theory. We have, however, made an attempt to extend the concept of modern filter theory, also, to these types of fluidic filters. Use of modern filter theory for design of fluidic filters gives us a more systematic synthesis approach. Through use of modern filter theory, we can design a filter to match the specified frequency response precisely; even the tolerances can be specified.

So far as distributed parameter filters are connected, there has been very little work in fluidics. The methods available are mostly analytical in which certain kinds of fluidic structures, which are known to give filtering effect,

are used. As an illustration of this approach, we have discussed the method proposed by A. Kohl [15].

4.1 LUMPED PARAMETER FILTER DESIGN:

We studied in Section 2.4 and Section 2.5 that if the length of a fluidic line is very small it can be treated as a lumped element so that a fluidic line with small diameter will give an inductance and a fluidic line with large diameter will give a capacitance which are given by equation (4.1).

$$L = \frac{\rho}{A} \ell \quad \text{(small diameter)} \qquad (4.1a)$$

$$C = \frac{A}{\rho a^2} \ell \quad \text{(large diameter)} \qquad (4.1b)$$

As a general rule the lumped parameter fluidic filters give a satisfactory result as long as their overall diamensions are less than quarter of the wavelength of the signal propagating through them.

Flanagan used this approach successfully to design the acoustic filters for digital communication circuits [16]. But he used the method of coefficient matching which was suitable for the perticular purpose. In general, however, the fluidic filters can be designed by using one of the two approaches which are i) image parameter technique and ii) modern filter theory.

We have presented the lumped parameter fluidic filter design in relation to the fluidic lines of circular cross-section which are most commonly used in practice.

4.1.1 Image Parameters Filter:

A fluidic filter can be realised from constant K filter design of an electrical filter . The series inductances and shunt capacitances of constant K filter can be realised through fluidic lines of small and large diameter put in cascade. The cascade arrangement of these fluidic elements give the effect of a ladder network as discussed in Section 2.5.2. We will present the design of image parameter fluidic filters through some examples given below.

<u>Example 4.1</u>:

Let us design a filter with a cut off frequency of 500 Hz and a load of $5 \times 10^5 \Omega$ at both ends. Filter with such specifications can be used in application like pressure smoothing in wind tunnel measurements [2]. The load impedance of this magnitude is typically encountered when source is the acrodynamic surface under test and load is a transducer arrangement.

The exact calculation of load and source impedance is quite complicated [17,18]. A rough estimate, however, can be found from the relation $R = \frac{\rho a}{A}$. Thus a load of 5×10^5--

will be caused by a fluidic element with a diameter of around 3.2 cm as clear from calculations given below:

$$R = \frac{\rho a}{A} \tag{4.2}$$

$$= \frac{\rho a}{\pi r^2}$$

$$\text{or,} \quad r = \sqrt{\frac{\rho a}{\pi R}}$$

$$= \frac{1.21 \times 340}{\pi \times 5 \times 10^5}$$

$$= 1.618 \text{ cm}$$

$$\text{or,} \quad d = 3.2 \text{ cm}$$

The element values of the T network can be obtained through the calculations given below

$$L = \frac{R}{\pi f_c} \tag{4.3}$$

$$\text{or,} \quad \frac{L}{2} = \frac{R}{2\pi f_c}$$

$$= \frac{5 \times 10^5}{2 \times \pi \times 500}$$

$$= 0.159 \times 10^3$$

and

$$C = \frac{1}{\pi f_c R} \tag{4.4}$$

$$= \frac{1}{\pi \times 500 \times 5 \times 10^5}$$

$$= 1.27 \times 10^{-9}$$

We assume that the diameter of the main inductive element is 1 cm and that of capacitive element is 4 cm. In practice these values can be chosen to suit the perticular requirement. The length of L and C elements can be calculated as

$$L = \frac{\rho}{A} \ell_1$$

$$\text{or, } \ell_1 = L \times \frac{A}{\rho} \qquad (4.5)$$

$$= \frac{0.159 \times \pi \times (0.5)^2 \times 10^{-4} \times 10^3}{1.21}$$

$$= 1.03 \text{ cm}$$

Similarily

$$C = \frac{A}{\rho a^2} \ell_2$$

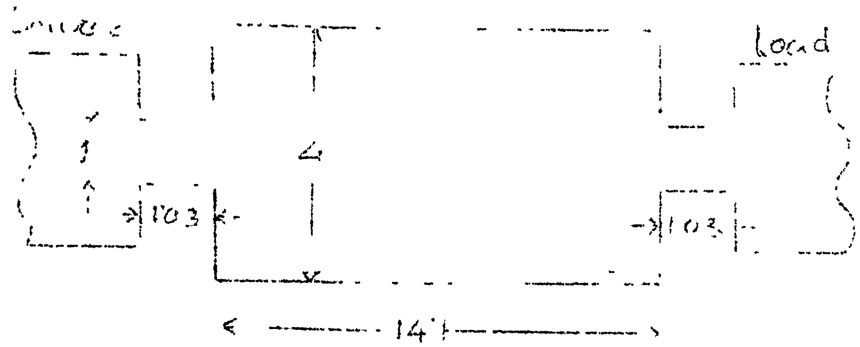
$$\text{or, } \ell_2 = C \times \frac{\rho a^2}{A} \qquad (4.6)$$

$$= \frac{1.27\times10^{-9}\times1.21\times(3.4)^2\times10^4}{\pi\times(2)^2\times10^{-4}}$$

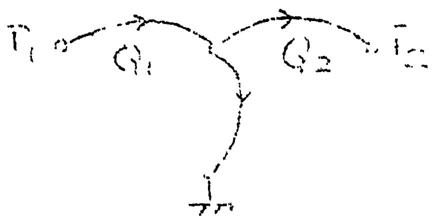
$$= 14.1 \text{ cm}$$

The filter configuration appears as given in Figure 4.1.

The filter obtained in the above example is not suitable for the type of application for which we want to use it. It will also give error since the overall length of the filter is very close to quarter of the wavelength.



(a) Fluidic filter



(b) System graph

Figure 4.1: Fluidic filter of example 4.1

The wavelength of the filter is given by,

$$\lambda = \frac{a}{f} = \frac{340}{500} = 68 \text{ cm}$$

However, the requirements of accuracy are not so stringent in case of silencers. It would seem, therefore, possible to use this kind of design in applications like silencers. The dimensions are also quite suitable to match the exhaust pipe. A cut off frequency of 500 Hz will give a good enough reduction in noise [17].

The situation is not always, so critical as it is in the previous example. For example if the load were to be 3 times more i.e. $15 \times 10^{-6}\Omega$, the length of C element would reduce 3 times, bringing a considerable reduction in overall length. Similarily a higher frequency will also result in smaller overall length. This can be seen from the example 4.2 given below.

Example 4.2:

We will now design the filter of example 1 with different specifications i.e. cut off frequency, $f_c = 1.5$ KHz and load impedance, $R = 1.5 \times 10^6 \Omega$. Let us assume the same condition of load and source arrangement as given in example 4.1. A load of $1.5 \times 10^6 \Omega$ will mean a load and source arrangement with a diameter of the order of 1.9 cms as given below

$$R = \frac{\rho a}{A}$$

$$\text{or,} \quad r = \sqrt{\frac{\rho a}{\pi R}}$$

$$= \frac{1.21 \times 340}{\pi \times 1.5 \times 10^6}$$

$$= 0.95 \text{ cm}$$

$$\text{or,} \quad d = 1.9 \text{ cm}$$

The element values of the T network are calculated below.

$$L = \frac{R}{\pi f_c} \quad \text{or} \quad \frac{L}{2} = \frac{R}{2\pi f_c}$$

$$\frac{L}{2} = \frac{1.5 \times 10^6}{\pi \times 1.5 \times 10^3}$$

$$= 0.159 \times 10^3$$

$$C = \frac{1}{f_c R}$$

$$= \frac{1}{\pi \times 1.5 \times 10^3 \times 1.5 \times 10^6}$$

$$= 1.414 \times 10^{-10}$$

If we again take the diameter of inductive and capacitive elements as 1 and 4 cm respectively, the length of these elements will be as given below.

$$L = \frac{\mu}{A} \ell_1$$

$$\text{or,} \quad \ell_1 = \frac{0.159 \times 10^3 \times \pi \times (0.5)^2 \times 10^{-4}}{1.21}$$

$$= 1.03 \text{ cm}$$

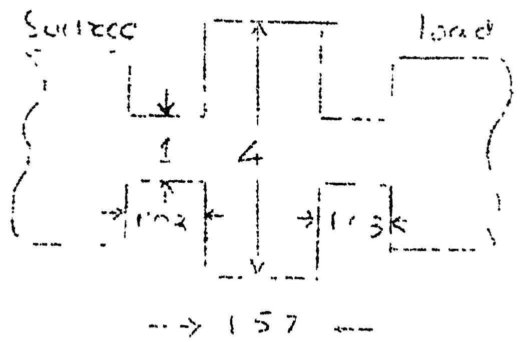
Also,

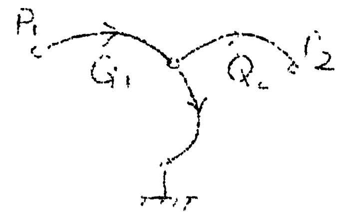
$$C = \frac{\epsilon A}{a^2} \ell_2$$

$$\ell_2 = \frac{1.414 \times 10^{-10} \times 1.21 \times (3.4)^2 \times 10^4}{\pi \times (2)^2 \times 10^4}$$

$$= 1.574 \text{ cm}$$

The filter configuration so obtained is given in Figure 4.4. This time the dimensions are quite satisfactory.





(a) Fluidic filter (b) System graph

Figure 4.2: Fluidic filter of example 4.2

The length of inductive elements can be further reduced if we reduce the area of cross section. A dia of 0.5 cm will give the length, ℓ_1 4 times less i.e. 0.25 cm. Similarily the length of capacitive element can be reduced by increasing the diameter.

What we have discussed above is design of a prototype low pass filter. In practice a number of such sections will be connected in cascade to meet the specified frequency response. Each section will decide a perticular frequency at which the attenuation is very high. The steepness of filter response will depend on number of sections used with each

section contributing to a roll off of 6 dB per octave. Thus the designed frequency response can be approached by using appropriate number of filter sections in cascade.

A major problem encountered in the image parameter filters is that of termination. We have seen in electrical context that for proper matching a constant K filter should be terminated in its characteristic impedance which varies continuously with the frequency. We cannot resolve this problem by using m derived section as we did for electrical filters. The m derived filter uses a combination of L and C in shunt arm which is difficult to realise with lumped fluidic elements. Use of modern filter theory gives a satisfactory answer to these questions.

4.1.2 Modern Filter Theory:

When a fluidic filter is to be designed to meet the detailed specifications like attenuation at different frequencies, use of modern filter theory gives more satisfactory results. Starting from given specifications, we can obtain a fluidic configuration that will meet the requirements exactly.

It has got an additional advantage that the filter so designed matches exactly to the given load impedance. Let us illustrate the procedure by taking some examples given below.

Example 4.3:

Let us design a 5th order Butterworth filter with cut off frequency of 1 KHz. We will assume that the filter is connected to a long tube of 1 cm diameter at both the ends. So that the source and load impedance can be approximated to be

$$R_S = R_L = \frac{\rho a}{A} = \frac{1.21 \times 3.40}{\pi \times (0.5)^2 \times 10^{-4}}$$

$$= 5.238 \times 10^6$$

The element values for the 5th order Butterworth filter are

$L_1 = 0.618$

$C_2 = 1.618$

$L_3 = 2$

$C_4 = 1.618$

$L_5 = 0.618$

The normalised values can be obtained by relations

$$L' = \frac{LR}{\omega_c}$$

$$C' = \frac{C}{\omega_c R}$$

$L_1 = 0.515 \times 10^3$

$C_2 = 0.0491 \times 10^{-9}$

$L_3 = 1.667 \times 10^3$

$$C_4 = 0.0491\text{x}10^{-9}$$

$$L_5 = 0.515\text{x}10^3$$

If we consider the diameter of L and C elements as 1 cm and 4 cms respectively, the length of each section will be given by

$$L = \frac{\rho}{A}\ell_L$$

$$\text{or,} \quad \ell_L = \frac{LA}{\rho} = \frac{L \times \pi \times r^2}{\rho}$$

$$C = \frac{A}{\rho a^2}\ell_C$$

$$\text{or} \quad \ell_C = \frac{C \times \rho \times C^2}{\pi \times r^2}$$

$$\ell_1 = 1.06 \text{ cms}$$

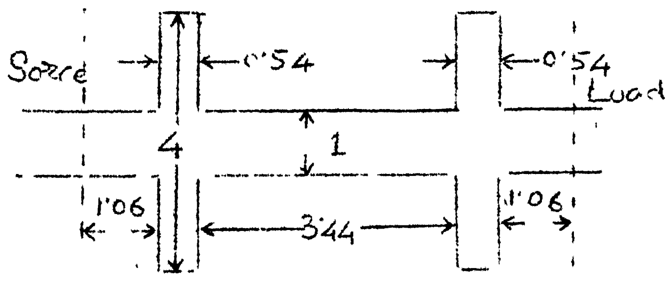
$$\ell_2 = 0.546 \text{ cms}$$

$$\ell_3 = 3.44 \text{ cms}$$

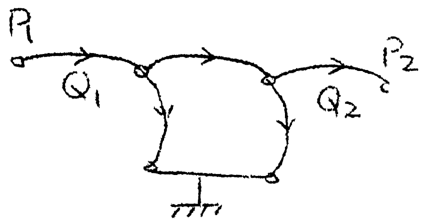
$$\ell_4 = 1.06 \text{ cms}$$

$$\ell_5 = 0.546 \text{ cms}$$

The overall design will emerge as given below in Figure 4.3.



(a) Fluidic filter



(b) System graph

The length of middle section can be reduced if the situation demands, by reducing the diameter of tube. If we reduce the diameter by a factor 2, the length will reduce 4 times.

Example 4.4:

We will now take up the design of a filter for unequal termination with $f_c = 3.4$ KHz. Let us assume that the source impedance is $5.238 \times 10^6 \Omega$ as in case of example 4.3 and the load impedance is considerably larger than the source impedance. Let us design a 4th order Butterworth filter.

From the table of Butterworth filter, we will select the element values for $\frac{R_L}{R_s} = \infty$, as given below.

$$L_1 = 1.53$$
$$C_2 = 1.57$$
$$L_3 = 1.08$$
$$C_4 = 0.38$$

The normalised values are given by

$$L' = \frac{LR}{\omega_c}$$

$$C' = \frac{C}{\omega_c R}$$

$$L_1 = 0.372 \times 10^3$$
$$C_2 = 0.14 \times 10^{-10}$$

$L_3 = 0.093 \times 10^3$

$C_4 = 0.096 \times 10^{-10}$

For diameters of 1 cms and 4 cms for L and C, the individual section will have lengths given by

$$L = \frac{\rho}{A} \ell_L$$

$$\text{or,} \quad \ell_L = L \frac{\pi r^2}{\rho}$$

$$C = \frac{\rho_A}{a^2} \cdot \ell_C$$

$$\text{or,} \quad \ell_C = C \frac{\rho a^2}{\pi x r^2}$$

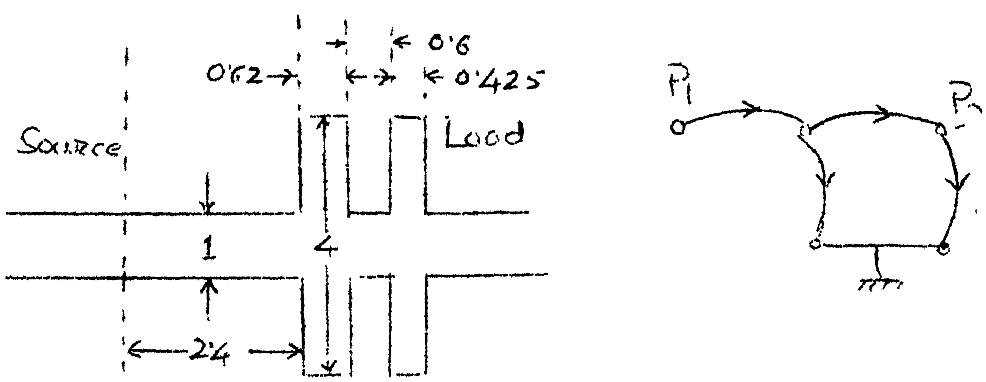
$\ell_1 = 2.41$ cms

$\ell_2 = 0.62$ cms

$\ell_3 = 0.6$ cms

$\ell_4 = 0.425$ cms

The filter dimensions will be as given in Figure 4.4.



(a) Fluidic filter (b) System graph

Figure 4.4: Fluidic filter of example 4.4.

If we want to install this filter in a small length, then we can reduce the length of element ℓ_1 by reducing its cross-sectional area. If we take a diameter of 0.5 cm in place of 1 cm; the length ℓ_1 will become 0.6 cms as given in Figure 4.5.

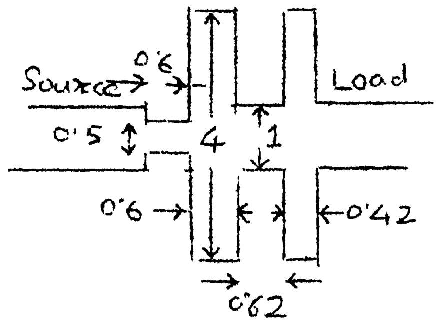


Figure 4.5: Modified design of example 4.4

This is a very useful design. It can be used for acoustic filtering of voice signal in a digital communication system. It can be conveniently installed in the headset with a slight modification.

Conventionally the digital communication system use electrical filtering for the band limiting the voice channels. J.L. Flanagan suggested that the overall cost can be considerably reduced if acoustic filters proposed by him are used in place of electrical filters [16]. His method, however, was analytical based on the open circuit response of a ladder network. We have used here a systematic approach based on modern filter theory.

4.2 DISTRIBUTED PARAMETER FILTERS:

Although the lumped parameter theory, provides a simple design technique, it imposes certain restrictions on the dimensions of fluidic filter. The actual filter performance will not match with theoretically predicted response as the overall dimension approach quarter of a wavelength, owing to the fact that for these proportion, the idealized models of the elements do not hold good.

To get over this difficulty, we have to treat the fluidic elements not in terms of lumped parameter elements but rather as elements with distributed parameters. But if we do so, using existing method of analysis to obtain an exact synthesis procedure becomes almost impossible owing to the difficulty in computations. It has been the practice in past to use certain fluidic line configurations which are known to give the filtering effect. The fluidic filter is derived from these configurations by analysing them and changing the dimensions till the fluidic system gives the nearest possible filter response.

Various types of arrangements have been tried for realisation of distributed parameter filters. Most interesting amongs them are the ones suggested by A. Kohl of Technischen Hochschule Aachen, Germany [15]. He used certain stub line configurations which are well known to acoustic

systems. Two basic configurations which he used in the design of low pass filters are shown in Fig. 4.8.

The element of Figure 4.6(a) is a simple stub line with uniform cross section. What is given in Figure 4.6(b) is the Helmholtz resonator used in acoustics. Such configurations are only possible with rectangular tubes since the ends of stub lines should physically match with the main fluidic line. Hence it is the common practice to use rectangular tubes in designing the distributed parameter fluidic filters.

The stub line of Figure 4.6(a) is treated as a transmission line open circuit at the far end since the closed end resembles a very high impedance. The stub line with stepped cross section of Figure 4.6(b) can be treated as cascade arrangement of two transmission lines of different characteristic impedances. Their frequency response can be calculated using equation (2.8) or by solving their matrix equations given by equation (2.32).

A Kohl performed experiments with the two stub line configurations and found the response to be in good approximation with the theoretical results. The frequency responses of these configurations are given in Figure 4.7. Therefore the analysis of the networks involving stub lines can be carried out by using matrix representation of each line

section. The overall system matrix can be obtained by multiplication. The response of the system can be found from this system matrix.

A fluidic filter is then, obtained by suitable placement of these stubs along a fluidic line. A typical low pass filter so obtained is given in Figure 4.8. The filter has got a uniform depth of 2.9 mm.

The filter of Figure 4.8 is designed for a cut off frequency of about 1.7 KHz. The design can be converted to any other desired frequency by changing the dimensions preportionally. Thus a low pass filter with cut off frequency of 3.4 KHz will have the length and width of the fluidic lines half of what is given in figure.

A. Kohl used this kind of filter in the fluidic speech transmission system and found the results to be quite satisfactory. However the method proposed by him does not give a straightforward synthesis procedure in which we can start from specifications and determine what sort of constructions will give the desired response. In the next section we will attempt to devise a procedure of synthesising fluidic filters using Richard's transformation discussed in Section 3.3.

SECTION 5

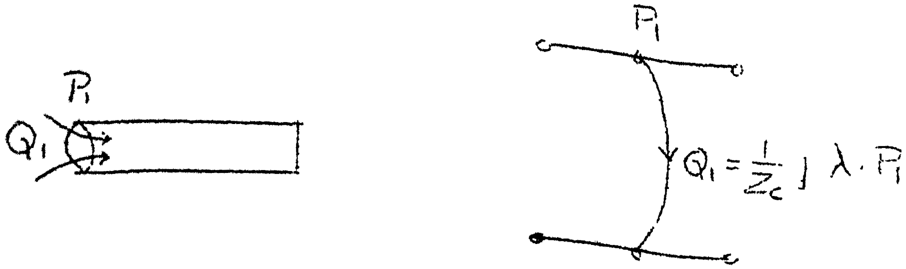
FLUIDIC FILTER DESIGN USING RICHARD'S TRANSFORMATION

We have seen in the preceeding section that stub line configurations closed or open ended at the far end give a filtering effect. These closed and open ended lines have resemblance to open and short circuit transmission lines. It has also been seen in Section 3.3 that using Richard's transformation ($\lambda = \tanh \Gamma \ell$) the open and short circuit transmission lines can be viewed as capacitance with $C = \frac{1}{Z_o}$ or inductance with $L = Z_o$ respectively. This gives us an intutive feeling that Richard's transformation can be used for synthesis of fluidic filters.

5.1 THE GENERAL PRINCIPLE:

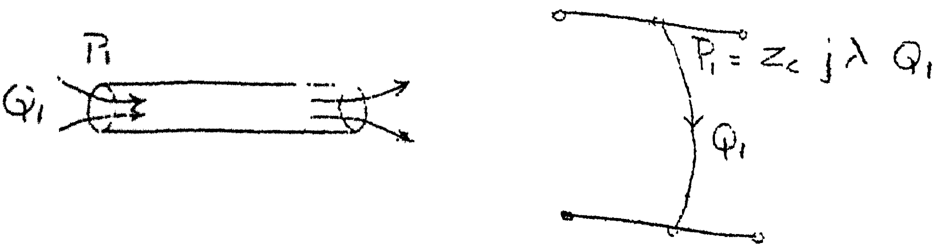
We will first examine the possibility of using the destributed parameter fluidic lines for realisation of fluidic filters. To begin with let us examine the behaviour of closed and open ended fluidic lines after having applied the Richard's transformation $\lambda = \tanh \Gamma \ell$. The system graph of Figure 2.15 for these two configurations reduce to two port elements as shown in Figure 5.1 and Figure 5.2 respectively.

Now, if we treat λ as an independent frequency variable, the system graph of a closed ended fluidic line resembles that of a shunt capacitance of an electrical system



(a) Fluidic line (b) System graph

Figure 5.1: System graph of a closed ended fluidic line after application of Richard's transformation.



(a) Fluidic line (b) System graph

Figure 5.2: System graph of an open ended fluidic line after application of Richard's transformation.

with $C = \frac{1}{Z_c}$. Similarily the system graph of an open ended line resembles a shunt inductance with $L = Z_c$. We have, thus found an effective way of realising fluidic inductance and capacitance with the help of open and closed ended fluidic lines.

There are certain difficulties arising out of the fact that both capacitive as well as inductive elements using closed and open fluidic lines can only be realized in ptractic as stub lines, behaving like the shunt elements. The first problem is that of interconnection of the stubs by appropriate

means. The second problem is that the standard low pass electrical filter network cannot be used for fluidic filters as such since it requires the use of inductive elements in series rather than shunt.

The problem of interconnection is sorted out by the use of unit element. In relation to fluidic systems, a unit element is a fluidic line of a predetermined fixed length. The length is predetermined to obtain a uniform delay.

All the unit elements and the stub lines will be of a length given by the frequency transformation $\lambda = \tanh \Gamma \ell$.

The second problem is that of getting a suitable electrical filter network which can be easily realised with fluidic elements. This network should not only have shunt elements alone but also account for the unit elements that are required to be inserted in between the elements. This is done by transforming the standard ladder network into a network in which shunt elements are separated by unit elements. This is achieved by first inserting the hypothetical unit elements at the source and load of the ladder network and then applying Kuroda's identity (discussed in Section 3.2) repeatedly to get the desirable network configuration. The unit elements are inserted in such a way that the loading conditions of the network do not change.

We have developed a design procedure based on the above principles in the following sections. Since the analysis underlying the design principle is essentially based on distributed parameter theory, we can put the filter design in the category of distributed parameter filter.

5.2 SYNTHESIS PROCEDURE:

The synthesis procedure can be explained in following steps [19].

i) Warp the frequency parameters using Richard's transformation, $\lambda = \tanh \Gamma \ell$. For a lossless fluidic line the propagation constant Γ is given by

$$\Gamma = j\beta = j\,\frac{\omega}{a}$$

$$\text{or,} \quad \lambda = \tanh j\frac{\omega}{a}\ell$$

$$= \tan \frac{\omega}{a}\ell$$

Thus the cut off frequency ω_c will be transformed into

$$\omega_c' = \tan \frac{\omega_c}{a}\ell \, .$$

where ℓ is the unit length i.e. the length of the UE as well as that of stub lines.

Generally from realisation point of view, unit length is chosen such that the transformed cut off frequency ω_c' is in the vicinity of 1. But sometimes the situation may arise

where this condition cannot be met . To appreciate this, let us examine the frequency response of a low pass filter given in Figure 5.3.

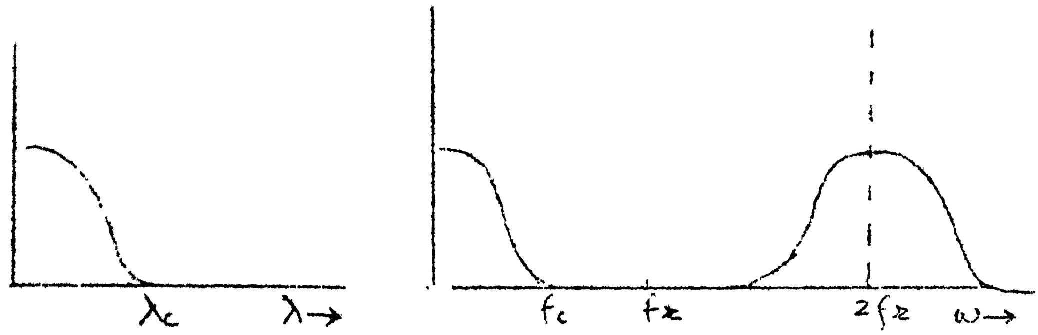


Figure 5.3: Frequency response of a low pass filter

We have seen in Section 3.3 that a low pass filter in λ domain acts as a comb filter in real frequency domain with the repeatition interval that is decided by ℓ . For the satisfactory performance, the frequency band of our interest should not extend beyond the rising edge of second pass band. When the required frequency band is large we would required the spread between the pass bands to be more. This will mean that the unit length, ℓ will have to be large as it is clear from the equation (5.1) which gives the reference frequency f_r (refer Figure 5.3).

$$\frac{\omega_r}{a}\ell = \frac{\pi}{2} \tag{5.1}$$

Thus if we want to increase f_r in relation to cut off frequency f_c, we will be required to have a large value of

This will have certain implications on the realizability of filter which has been discussed later in detail.

ii) Obtain suitable ladder network from the table of electrical filters.

Depending upon the specifications like steepness, ripple, delay, a suitable filter approximation can be considered from the variety of filter configurations available. Then an appropriate design is selected from the table depending upon the ratio of source to load impedance and the order of filter required. All filter specifications like steepness, cut off frequency etc. will be taken in transformed frequency domain. The design will be normalised to the required cut off frequency ω'_c and the load impedance. The normalised element values will be

$$L' = \frac{LR_L}{\omega'_c} \qquad C' = \frac{C}{\omega'_c R_L}$$

where R_L is the load impedance and ω'_c is the transformed cut off frequency.

iii) Insert appropriate number of UEs at source and load terminals.

This is an hypothetical situation since UEs have got no significance in terms of lumped electrical elements. We use this concept to modify the ladder network so as to make it realisable through fluidic line elements. Since the UE is

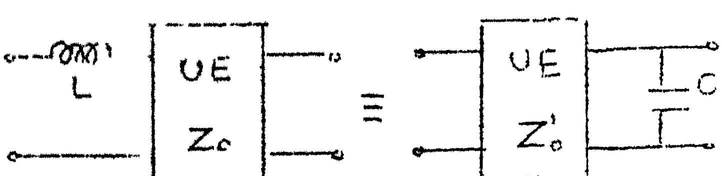
basically a section of fluidic line, it can be thought of as a symmetrical T network in terms of lumped representation. The overall loading conditions of the network, therefore, will not change if the impedance in which they are terminated is same.

The number of UEs at each end will depend upon the order of filter. Itw will be selected such that the final design (after applying Kuroda's identity) will have shunt elements separated by UEs. This can, often, be achieved in number of ways we will select a combination which will require least number of Kuroda's identities to be applied to get the desired filter configuration. This is important from realisation point of view. However if circuit is singly terminated then all the UEs will have to be inserted on one side.

iv) Apply Kuroda's identity repeatedly to obtain a design with shunt elements separated by UEs.

As we have seen earlier such an arrangement is desirable from realisation point of view. For low pass filter, the two types of identities mostly used are given in Figure 5.4.

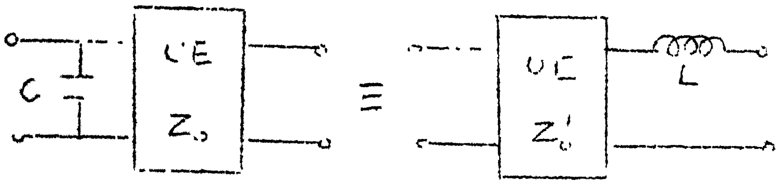
(a)



$$Z_o' = Z_o + L$$

$$C = \frac{L}{Z_o Z_o'}$$

(b)



$$Z_o' = \frac{Z_o}{1+Z_oC}$$

$$L = Z_oC\,Z_o'$$

Figure 5.4: Two most useful forms of Kuroda's identity

v) Calculate the cross sectional area of all the sections from their characteristic impedance using relation $Z_o = \frac{\rho a}{A}$.

The characteristic impedance of stub lines will be given by $Z_o = L$ or $Z_o = \frac{1}{C}$ depending upon whether it is inductive or capacitive. An inductance will be realised through open ended stub line of unit length whereas a capacitance will be realised through closed ended stub line of unit length.

Following the above synthesis procedure, we will invariably get a configuration in which the cross sectional area of stub lines is more than the cross sectional area of the UE connecting them. It is physically impossible to realise such an arrangement through circular tubes. Therefore, we will use rectangular tubes for realisation of fluidic filters. All the filter elements will have a uniform depth and their width will be decided by the cross secctional area required.

We will, now, consider some practical examples to illustrate the procedure mentioned above.

Example 5.1:

Let us design a fluidic filter with a cut off frequency f_c = 1000 Hz.

A unit length, ℓ = 4 cm gives ω'_c as

$$\omega'_c = \tan \frac{\omega}{a} \ell$$

$$= \tan \frac{2\pi \times 1000 \times 4 \times 10^{-2}}{340} \times \frac{180^\circ}{\pi}$$

$$= 0.9116$$

Let us assume that the filter is terminated in infinite exponential horms at both the ends. With a cross sectional area of 5x5 mm at the throat. It is reasonable to assume a cross section of 5x5 mm for a fluidic line with unit length of 4 cm. Such an arrangement is convenient for testing the filter with the acoustic signal. The practical situation may differ but the magnitude of impedance will be of the same order as encountered in the present case.

The impedance of exponential horn is given by the expression [18]

$$R_L = \frac{\rho a}{A_T}$$

where A_T is the cross sectional area at the throat

$$R_L = \frac{1.21 \times 340}{5 \times 5 \times 10^{-6}} = 1.6456 \times 10^7$$

Let us consider a third order of Butterworth filter for the sake of simplicity. The ladder network obtained from the table of Butterworth filters is given in Figure 5.5.

The normalised values of filter element will be

$$L' = \frac{LR}{\omega'_c} = 1.805 \times 10^7$$

$$C = \frac{e}{\omega'_c R} = 1.333 \times 10^{-7}$$

Now we will introduce one UE each at source and load terminals. The characteristic impedance of these UEs will be $Z_o = 1.6456 \times 10^7 \Omega$. The resultant circuit is given in Figure 5.6.

After applying Kuroda's identity, the final arrangement will appear as given in Figure 5.7 with the following values of the parameters.

$$Z_{oT} = Z_o + L \quad \text{and } C = \frac{L}{Z_o Z_{oT}}$$

This gives

$$Z_{o1} = Z_{o2} = 3.45 \times 10^7$$
$$C_1 = C_3 = 0.318 \times 10^{-7}$$
$$C_2 = 1.333 \times 10^7$$

Let us assume a uniform depth of 5 mm throughout.

Then,

$$\text{width of UEs} = \frac{\varrho_a}{Z_c \times d}$$
$$= 2.3844 \text{ mm}$$

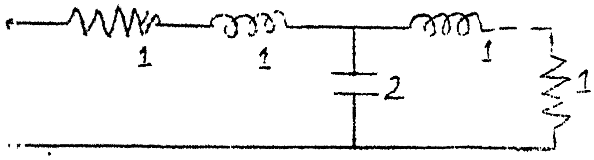


Figure 5.5: A third order low pass filter of Example 1

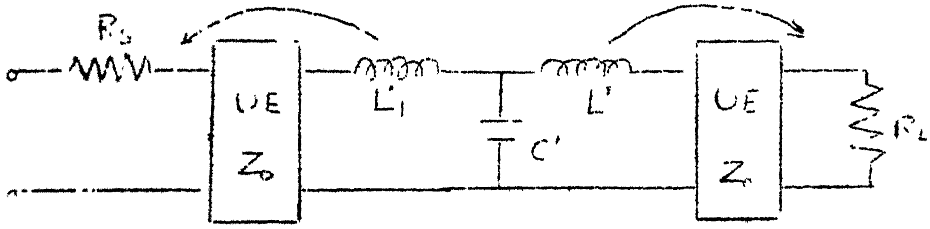


Figure 5.6: Inserting UEs near source and load terminals

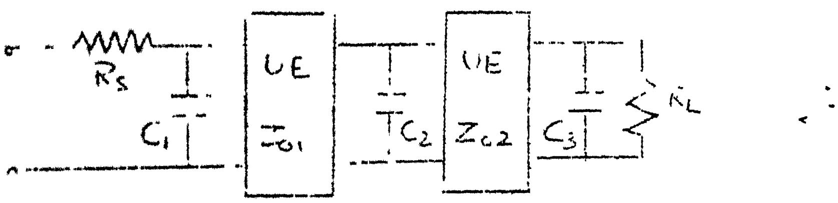
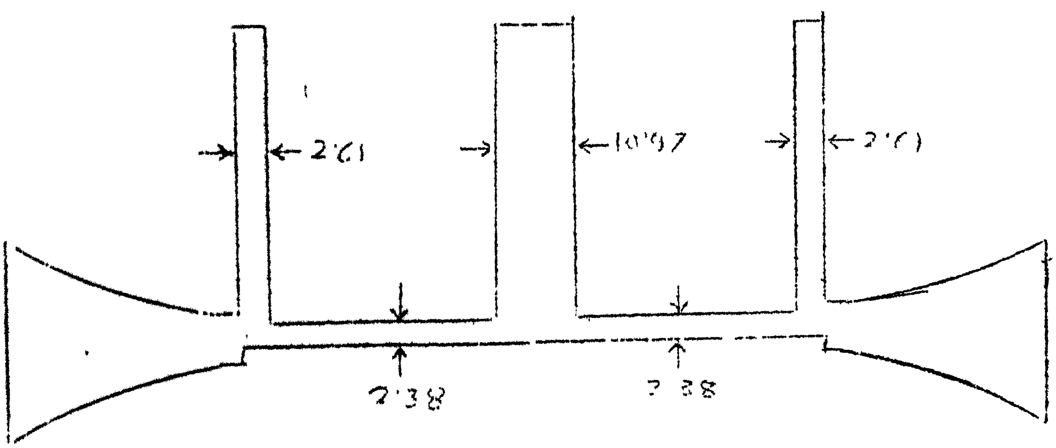
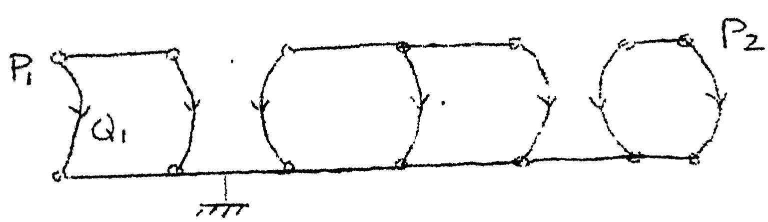


Figure 5.7: Filter configuration after applying Kuroda's identity



(a) Fluidic filter



(b) System graph

Figure 5.8: Fluidic filter of example 5.1. (The width of each section is indicated in mms. The unit length is 40 mm).

Similarly,

$$\text{width of stub lines} = \frac{\rho a}{Z_c \times d}$$

$$\text{where } Z_c = \frac{1}{C}$$

width of side stub lines = 2.615 mm and

width of centre stub line = 10.97 mm

The ultimate filter design will appear as given in Figure 5.8.

This arrangement is of practical significance since it can be easily fabricated in workshop and very conveniently tested with acoustic signals in the laboratory.

Example 5.2:

Let us consider the acoustic filter discussed in example 4.4 as another example. In section 4.1 we designed this filter using lumped parameter technique. We will now design the same filter using distributed parameter theory.

The cut off frequency is 3.4 KHz.

We have already seen that such a filter is of immense use in digital communication circuits for band limiting the speech signals.

For this purpose, normally, a Third Order Butterworth filter will meet the requirement more than adequately. We can go for even higher order filters but that will increase

the overall size since the number of elements will increase. Therefore we will consider a 3rd order filter for realisation.

Let us take a unit length, $\ell = 1.5$ cm

$$\text{then } \omega_c' = \tan \frac{2\pi \times 3.4\times10^{3}\times1.5\times10^{-2}}{340} \times \frac{180^\circ}{\pi}$$

$$= 0.9424$$

A small exponential horn can be put at the input so that the source impedance can be approximated to be

$$R_S = \frac{\rho a}{A}$$

$$= \frac{1.21\times340}{2\times2\times10^{-6}} = 1.103\times10^{8}$$

where A = 2x2 mm is the cross section of input horn at its throat. This is taken keeping in mind the uniform depth of 2 mm for the filter elements.

For sake of convenience in design let us take the load impedance to be equal to source impedance i.e. $1.103\times10^{8}\,\Omega$.

The normalised element values will be

$$L' = \frac{L}{\omega_c'} R = 1.17\times10^{8}$$

$$C' = \frac{C}{\omega_c'} \times \frac{1}{R} = 1.924\times10^{-8}$$

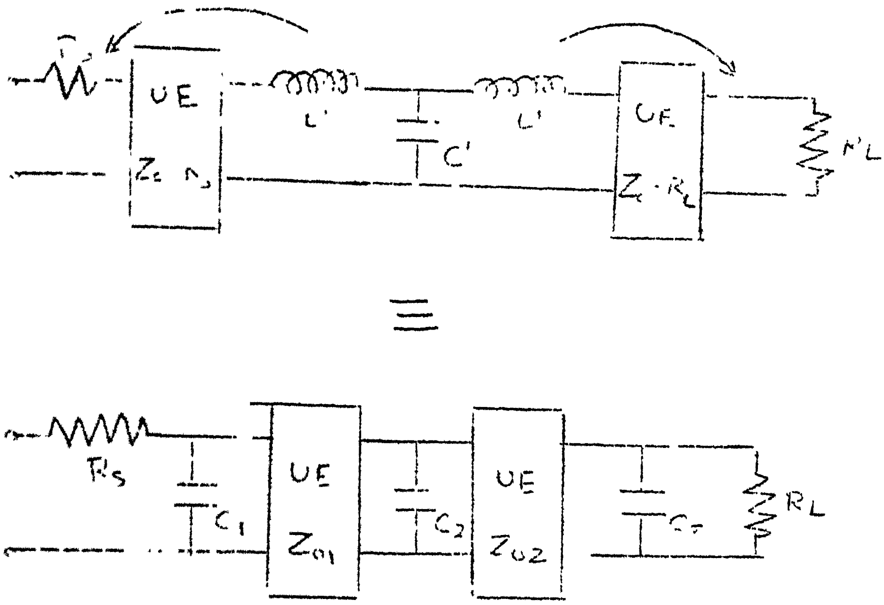
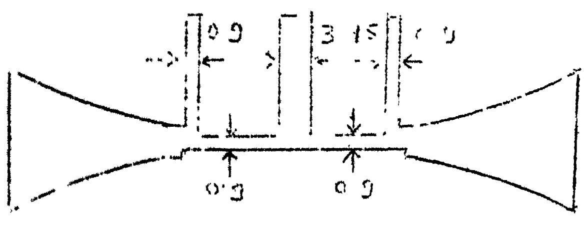
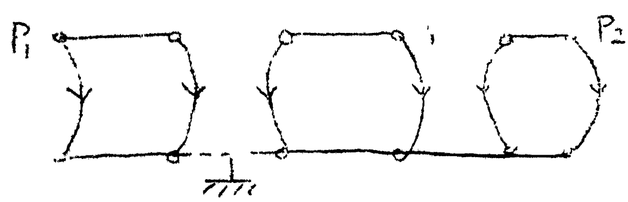


Figure 5.9: Applying Kuroda's identity to a third order filter.



(a) Fluidic filter



(b) System graph

Figure 5.10: Fluidic filter of example 5.2.(The width of each section is indicated in mms. The unit length is 1.5 cm).

Inserting UEs and applying Kuroda's identity as illustrated in Figure 5.9, we get

$$Z_o = 2.273\text{x}10^{8}$$
$$C_1 = C_3 = 0.466\text{x}10^{-8}$$
$$C_2 = 1.924\text{x}10^{-8}$$

Considering a uniform width of 2 mm we can calculate the width of various sections as

$$\omega_{UE} = 0.905 \text{ mm}$$
$$\omega_{c_1, c_3} = 0.958 \text{ mm}$$
$$\omega_{c_2} = 3.95 \text{ mm}$$

The design will appear as given in Figure 5.10.

The overall size of the filter is such that it can not be installed in a headset. It can, however, be used if the receiver is of bigger dimension like microphone.

5.3 PRACTICAL ASPECTS OF FILTER DESIGN:

In this section we will discuss the problems encountered in the implementation of given procedure and the ways and means of dealing them under different conditions. Some of these problems have already been encountered in the above sited examples; we will discuss their implications in detail.

A number of filter designs with different specifications have been studied for this purpose, using computer program

(PASCAL) given in Appendix 'D'. Some of the comments given below are based on the interpretation of these results.

5.3.1 Order of Filter:

We have considered a simple 3rd order filter in the examples of preceeding section. Higher order filters can be obtained similarily wherever required. But as we go for higher order filters more number of stub lines are required for realisation. This means more number of UEs needed and accordingly Kuroda's identity to be applied more number of times.

In low pass filter design, the cross sectional area of elements keep decreasing with the application of Kuroda's identity. Thus for the higher order filter the spread of values of elements will be very large. A seventh order Butterworth filter with a cut off frequency of 1 KHz is given in Figure 5.11, to give an idea of this fact.

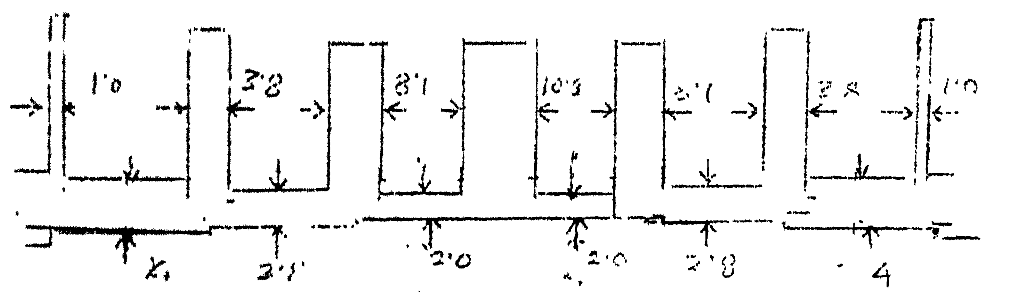


Figure 5.11: Seventh order Butterworth filter with f_c= 1 KHz. (The figure indicate the width of line section in mm . The unit length is 4 cms).

We have taken a unit length of 4 cm and the a uniform depth as 5 mm same as that of example 5.1, just to be able to compare the results. Here we see that the width of stub lines at the extreme ends is 1.0 mm as compared to 2.61 mm for a third order filter with same cut off frequency. A width of 1.0 mm is not only critical from fabrication point of view but also gives an aspect ratio of about 5. A high aspect ratio will give high attenuation of the signal. This is clear from the expression for the resistence per unit length of a rectangular tube i.e. $R = \frac{12\mu}{A^2}(\sigma + \frac{1}{\sigma})$ where σ is the aspect ratio. It is therefore, preferable to meet the requirement of the filter specification with the lower order filter design as far as possible.

5.3.2 Unit Length:

Selection of unit length is another critical point in fluidic filter design. If we take a very small value, the width of central stub will be very large, sometimes unrealisable. On the other hand if the unit length is taken very large, the width of extreme end stub lines may be extremely small.

Appendix 'E' gives the dimensions of a seventh order filter with a cut off frequency of 1 KHz, by taking various values of unit length. We see that the most reasonable design results by taking unit length of 4 cm which corresponds

to transformed cut off frequency $\omega_c' = 0.9116$. As a guide line it may be stated that unit length should be chosen so as to give ω_c' in the vicinity of 1.

5.3.3 Number of Unit Elements:

The number of UEs to be inserted at the source and load terminals should be chosen carefully, so as to result into a design with stub lines separated by UEs. This can be illustrated by taking example of 5th order filter given in Figure 5.12.

We first introduce one UE each at both the terminal and apply Kuroda's identity to get the filter design of Figure 5,13. To further simplify it we insert one UE each at both ends again and get a design given in Figure 5.14. This configuration has got series inductance and hence cannot be practically realised. We can further modify the circuit by inserting one UE each at both the ends again. For this we will have to break the middle inductor into two halves in a bid to transfer it to either side. By doing so we will ultimately get middle section with 2 UEs in series as given in Figure 5.15. This section will break the uniformity as it will introduce double the delay introduced by other sections.

A satisfactory design results by inserting 3 UEs at the load terminal and one UE at the source. This arrangement is shown in Figure 5.16.

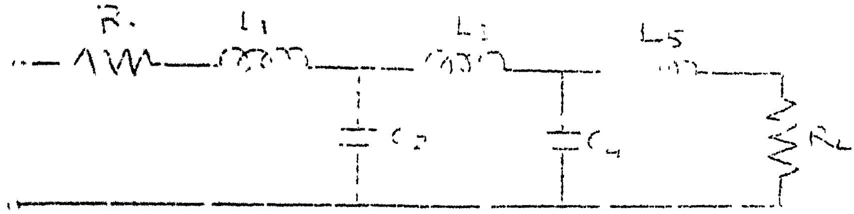


Figure 5.12: A fifth order filter network

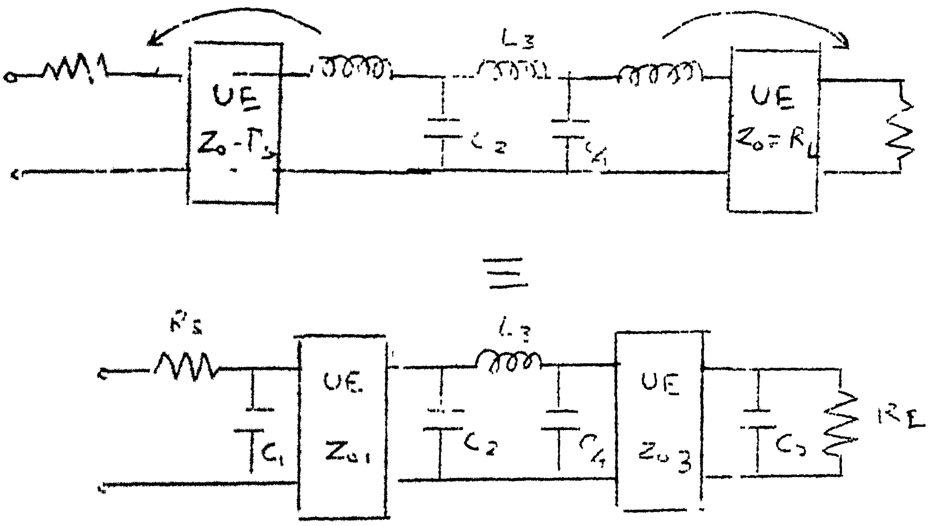


Figure 5.13: Fifth order filter after inserting 1 UE each at source and load terminals.

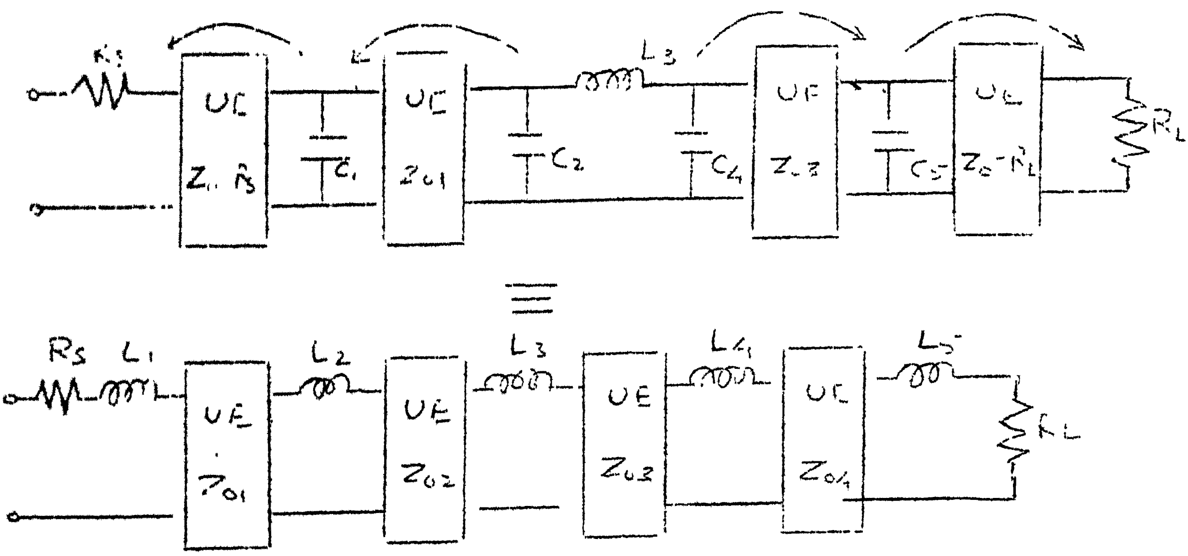


Figure 5.14: Fifth order filter after inserting 2 UEs each at source and load terminals.

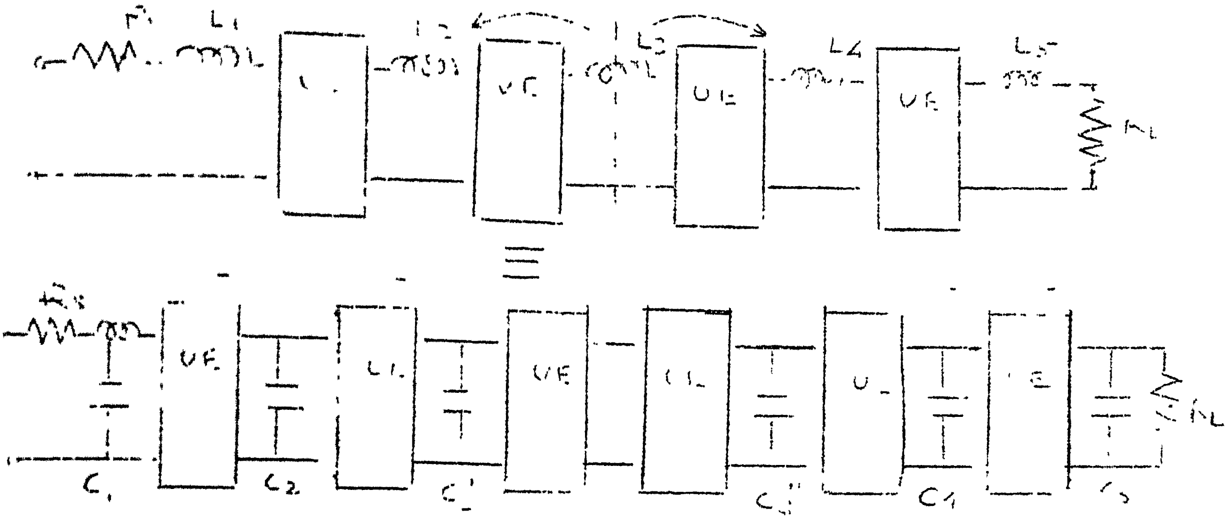


Figure 5.15: Simplifying the 5th order filter by inserting 3 UEs each from both the terminals.

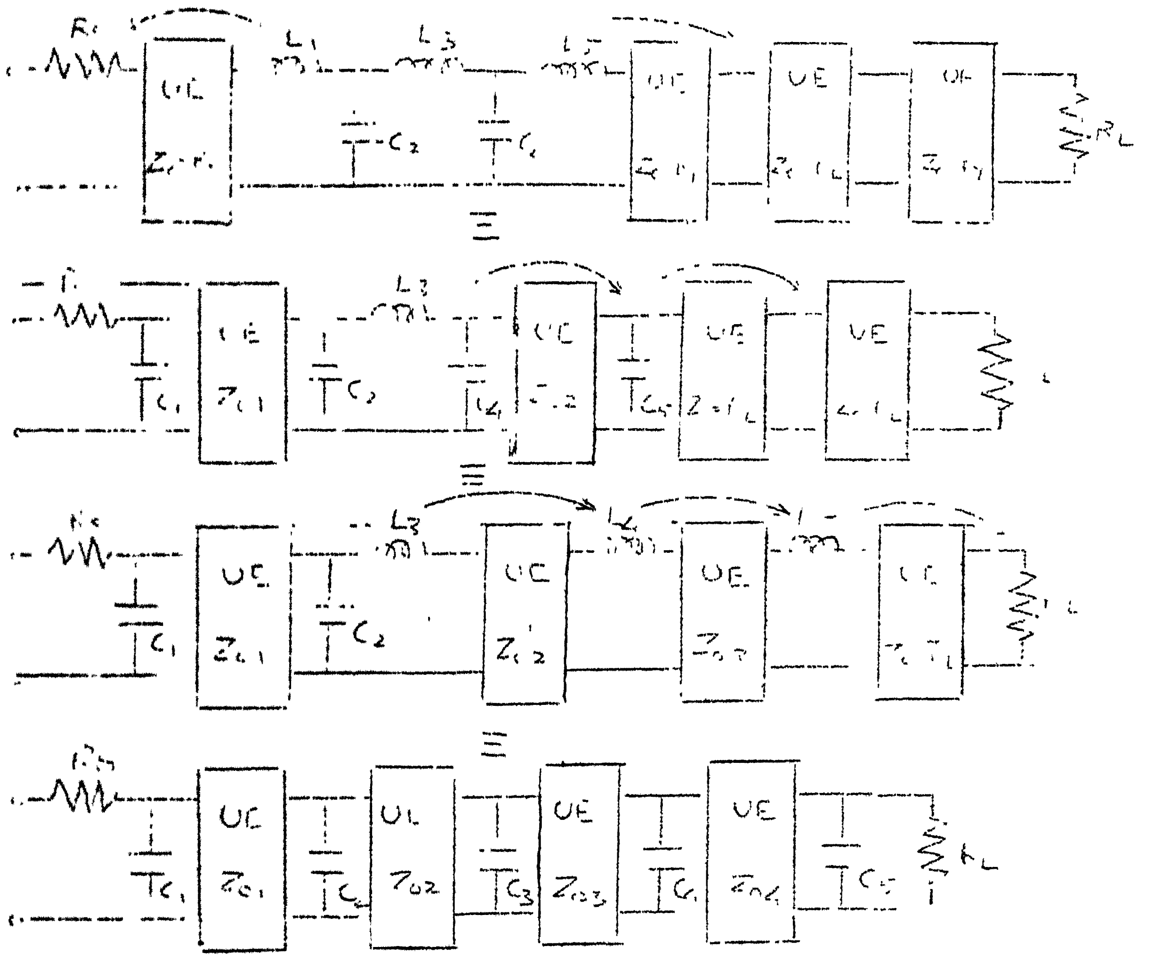


Figure 5.16: Design of 5th order filter by insertine one UE at source and 3 UEs at load terminal.

It may be pointed out that we could realise a desired configuration by inserting all the UEs on any one side of the circuit. But the arrangement of Figure 5.16 is more satisfactory since it involves 4 UEs in all as against 5 UEs otherwise required. More number of UEs will mean Kuroda's identity to be applied more number of times and more possibility of getting small cross sectional area increasing attenuation and posing problems in fabrication.

Table 5.1

Number of UEs to be used in fluidic filter design

Sl No.	Order of filter	No. of UEs at the source	No. of UEs at the load
1	1	0	0
2	2	1	0
3	3	1	1
4	4	1	2
5	5	1	3
6	6	1	4
7	7	3	3
8	8	3	4
9	9	3	5
10	10	3	6
11	11	5	5
12	12	5	6
13	13	5	7
14	14	5	8

Table 5.1 lists down the value of number of UEs that should be inserted on either side for the optimum results. The values have been listed for different order of filter. Based on the above results we have found out an empirical formula given below.

No. of UEs at the source, is equal to the integer part of expression $[2x(n-1)/4]-1$

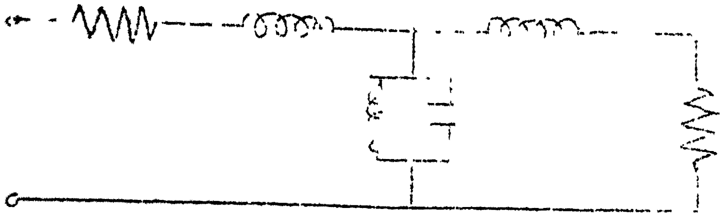
No. of UEs at the load = (n-1)- No. of UEs at the source.

It is this empirical formula which has been used in the program of Appendix 'D'.

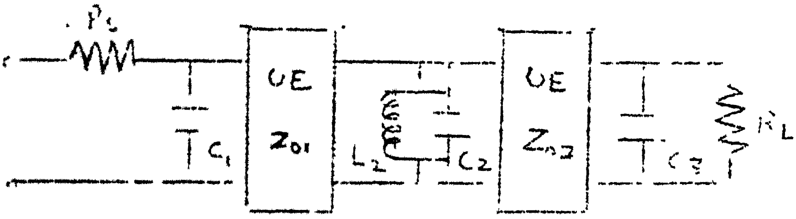
5.3.4 Elliptic Filter Design:

All types of low pass filters, except elliptic filters, use series inductances and shunt capacitances. This is very convenient scheme for applying Kuroda's identity to get a stub line filter. The situation is not same with elliptic filters which use resonant circuits in shunt arm. This type of arrangement gives a complicated structure through use of Kuroda's identity as seen from the Table 3.1.

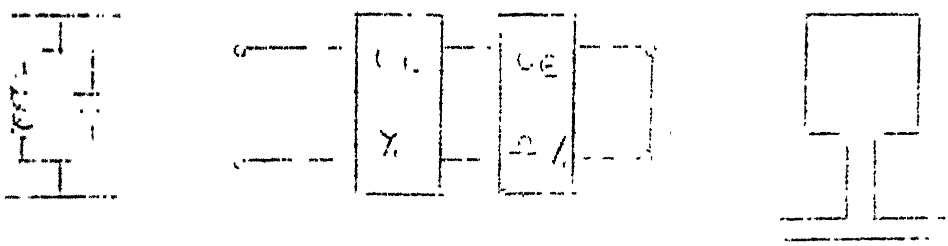
In general, elliptic filters cannot be straightaway converted to fluidic design easily. However simple configurations like a third order filter can be realised without much difficulty as shown in Figure 5.17. The resonance circuit



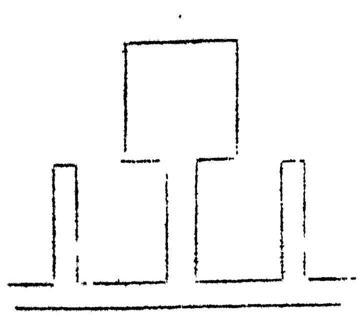
(a) 3rd order elliptic filter



(b) Third order elliptic filter after applying Kuroda's identity



(c) Realisation of a parallel resonance circuit.



(d) Third order elliptic fluidic filter

Figure 5.17: Design of a 3rd order elliptic filter.

is realised by putting 2 UEs in cascade as discussed in Section 3.3. The physical configuration of the resonance circuit comes out to be similar to a Helmholtz resonator [15].

5.3.5 Consideration of Losses:

There are mainly two types of losses associated with a fluidic system i) loss due to wall shear which can be expressed as resistence given by $R = \frac{12\mu}{A^2}(\sigma + \frac{1}{\sigma})\ l$, ii) loss due to stepped cross section and branchings which are given by drop in pressure expressed as $P_1 - P_2 = \frac{\rho u^2}{2}$ where u is the average velocity. In a practical system the losses associated with the branchings and sudden changes in cross sections are much smaller then the losses introduced by wall shear.

Let us consider the losses due to wall shear first which is given by resistence R.

$$R = \frac{12\mu}{A^2}\ (\sigma + \frac{1}{\sigma}) \times \ell$$

We will consider the dimensions of filter of example 5.1 to give an idea of the order of resistence normally encountered.

In example 5.1 the maximum loss will occur in UEs which has got least cross-sectional area of 5x2.38 mm.

$$R = \frac{12x1.8x10^{-5}}{(5x2.38x10^{-6})} \times (2.576) \times 0.04$$

$$= 0.0158 \times 10^{7}$$

This is much smaller then either reactance or load impedance as clear from the values of C_1 and R_L given below.

$$C_1 = 0.318\times10^{-7}$$

and $$R_L = 1.6456\times10^{7}$$

Thus we see that the resistence can be neglected without causing any significant error.

Coming back to the losses of second kind which are introduced by branching and stepped cross-sections we know that these losses are given by expression

$$P_1 - P_2 = \frac{\rho u^2}{2}$$

These losses are not serious in nature since their effect is equivalent to a constant pressure drop. Also the magnitude of these losses are much smaller than the losses due to wall shear as calculated above. To give a comparative idea of the magnitude we manipulate the above expression to get a rough idea of resistence values associated.

$$P_1 - P_2 = \frac{\rho u^2}{2}$$

but $$u = \frac{Q}{A}$$

$$\frac{P_1 - P_2}{Q} = \frac{\rho u}{2A}$$

$\frac{P_1-P_2}{Q}$ can be replaced by R interms of electrical analog.

$$R = \frac{\rho u}{2A}$$

From this expression it is clear that even under worst possible conditions i.e. when velocity of propagation is very high, the loss associated in terms of resistence in this case is much less than that due to wall shear.

Hence we see in general that the losses associated with the fluidic filter are very small compared to load impedance or . reactances and can be neglected for all practical purposes.

5.4 TABLE OF ELEMENT VALUES FOR FLUIDIC FILTERS:

A useful feature, of the synthesis procedure, discussed above is the fact that we can easily contruct tables of element values for normalized design and obtain element values for general specification from these through simple formulae. The readymade tables prove very handy in the fluidic filter design since they require fewer calculations [20].

For the purpose of constructing these tables we will consider the transformed cut off frequency ω_c' to be equal to unity. ω_c' will remain equal to 1 for all the filter designs; it is the unit length which will change as per the actual cut off frequency. The has been done because of two reasons. Firstly, it will simplify the design procedure which could have been very complex otherwise since every value of ω_c' will lead to unique values of filter elements. Second reason is that the spread of element values will be much less.

The element values given in the table are meant for load impedance of unity. The actual values can be calculated by normalising them to the required load impedance by relation $Z'_o = R_L Z_o$ or $C' = \frac{C}{R_L}$. The dimensions of the filter can be found using relation $Z_o = \frac{\rho a}{A}$.

Based on the principles discussed above a Pascal program has been developed which converts a table of electrical filter to a table of fluidic filters. The program is given in Appendix 'F'.

Appendix 'G' gives a table of element values for fluidic filter of Butterworth kind. We have chosen Butterworth filter since it is most simple and has been used in the examples throughout this text. Similar tables can be worked out for other kinds of filters like Chebychev, Bessel etc. There is no difference in either the method to work out these table or to use them in design.

We will consider an example to demonstrate the use of the tables.

Example 5.3:

Let us design a 5th order Butterworth filter with $f_c = 500$ Hz for a load of $R_s = R_L = 4.114 \times 10^6$. Such filters are useful in pressure smoothing in wind tunnel measurements as we have seen in example 4.1.

The unit length ℓ , is given by

$$\omega_c' = \tan \frac{\omega}{a} \ell$$

or, $$1 = \tan \frac{\omega}{a} \ell$$

or $$\frac{\pi}{4} = \frac{\omega}{a} \ell$$

or $$\ell = \frac{\pi}{4} \times \frac{a}{2\pi \times 500} = 8.5 \text{ cm}$$

The element values obtained from the table of Appendix 'G' are given below.

$C_1 = 0.381$

$ZC_2 = 1.618$

$C_3 = 1.617$

$ZC_4 = 2.447$

$C_5 = 1.827$

$ZC_6 = 1.894$

$C_7 = 0.854$

$ZC_8 = 1.276$

$C_9 = 0.216$

The normalised values are obtained from $Z_o' = R_L Z_o$ and $C' = \frac{C}{R_L}$

$C_1 = 0.0926$

$ZC_2 = 6.65 \times 10^6$

$C_3 = 0.393 \times 10^{-6}$

$$ZC_4 = 10.066\times10^6$$

$$C_5 = 0.444\times10^{-6}$$

$$ZC_6 = 7.792\times10^6$$

$$C_7 = 0.207\times10^{-6}$$

$$ZC_8 = 5.249\times10^6$$

$$C_9 = 0.0525\times10^{-6}$$

A load of $4.114\times10^6\,\Omega$ can be obtained by rectangular tube of cross sectional area 1x1 cm terminated into its characteristic impedance.

$$R_L = \frac{\rho a}{A} = \frac{1.21\times340}{1\times1\times10^{-4}} = 4.114$$

This gives us an idea of the depth of filter configuration.

Thus assuming an uniform depth of 1 cm we can calculate the width of various elements by expression

$$Z_o = \frac{\rho a}{A} \quad \text{or} \quad \frac{1}{C} = \frac{\rho a}{A}$$

$$\text{width} = \frac{a}{Z_o\times1\times10^{-2}}$$

$C_1 \rightarrow$ 0.381 cms

$UE_1 \rightarrow$ 0.618 cms

$C_2 \rightarrow$ 1.617 cms

$UE_2 \rightarrow$ 0.408 cms

$C_3 \rightarrow$ 1.827 cms

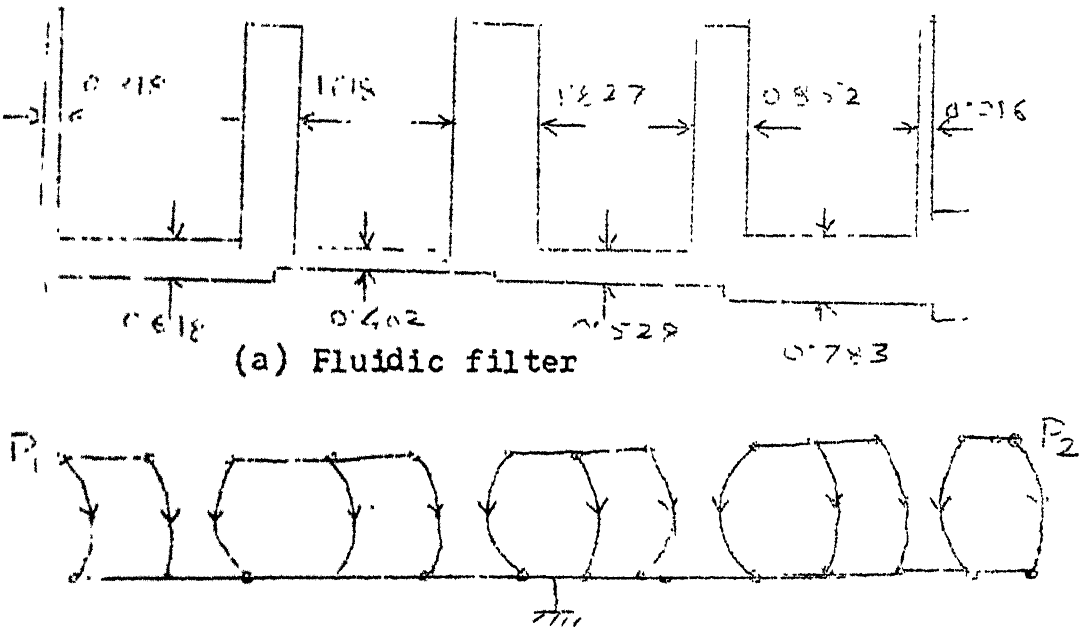
$UE_3 \rightarrow 0.528$ cms

$C_4 \rightarrow 0.853$ cms

$UE_4 \rightarrow 0.783$ cms

$C_5 \rightarrow 0.216$ cms

The filter will appear as given in Figure 5.18.



(a) Fluidic filter

(b) System graph

Figure 5.18: Flucidic filter design of example 5.3. (The width of each section is indicated in cms. The unit length is 8.5 cms).

Thus we see that a 5th order filter, which would have required enormous calculations by direct synthesis, has been obtained very easily by use of the ready-made table. The procedure given in example 5.3 look a little cumbersome since it is explainatory. In practice, however, all the calculations can be done side by side in a single step.

5.5 DESIGN OF HIGH PASS, BAND PASS AND BAND STOP FILTERS:

One of the most attractive features of the modern electrical filter theory is the frequency transformation through which low pass filter can be transformed into high pass (HP), band pass (BP) or band stop (BS) filter. All the arguments developed for low pass filters can be easily extended to HP, BP or BS filters through the transformations given in Table 5.2 [21].

Table 5.2

Frequency transformations of filters

S.No.	Type of filter	Transformation
1	LP to HP	$S \rightarrow \frac{1}{S}$
2	LP to BP	$S \rightarrow S + \frac{\omega_o^2}{S}$
3	LP to BS	$S \rightarrow \frac{S}{S^2+\omega_o^2}$

(ω_o is the resonance frequency)

These transformations allow us to convert an LP filter to HP, BP or BS filter through changes in electrical configuration as given in Figure 5.19.

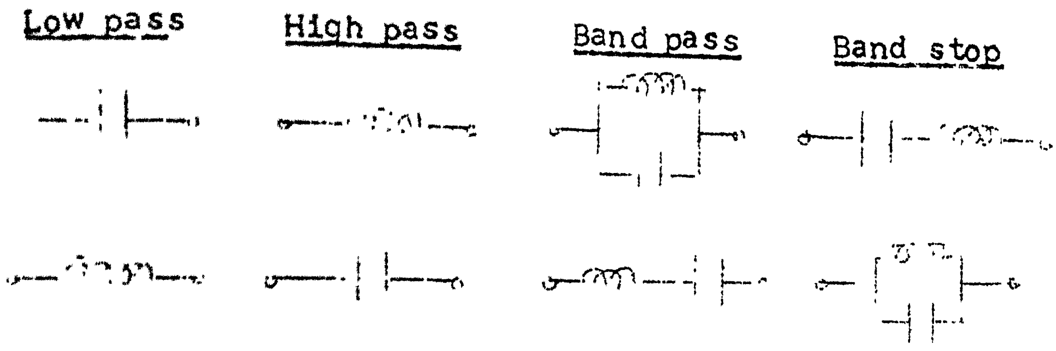
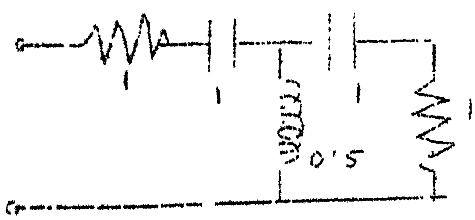
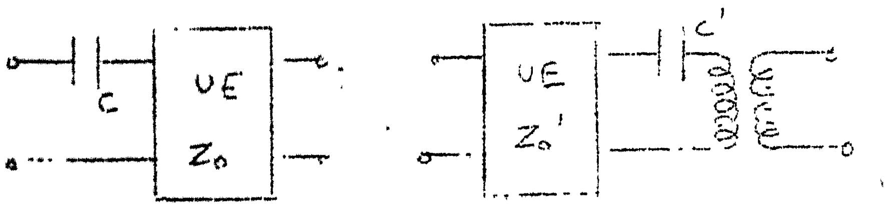


Figure 5.19: Frequency transformation of LP filter into HP, BP or BS filter.

Unfortunately these transformations cannot be applied to fluidic filters because the resultant filter configuration may not always be realisable. Let us consider the example of a simple 3rd order Butterworth filter to illustrate the fact (Fig. 5.20). The HP filter obtained after transformation is given in Fig. 5.20(a). If we try to apply Kuroda's identity to this circuit, we get a transformer which cannot be realised easily in fluidic systems.



(a) 3rd order HP Butterworth filter



(b) Required Kuroda's identity

Figure 5.20: Applying Kuroda's identity to 3rd order high pass filter.

An interesting possibility of frequency transformation exists in microwave filters which can be subjected to $\lambda \rightarrow \frac{1}{\lambda}$ transformation through use of ideal gyrators [12].

The transfer matrix of UE us

$$(1 - \lambda^2)^{-\frac{1}{2}} \begin{bmatrix} 1 & \lambda Z_o \\ \lambda / Z_o & 1 \end{bmatrix}$$

After applying transformation $\lambda \rightarrow \frac{1}{\lambda}$ we get

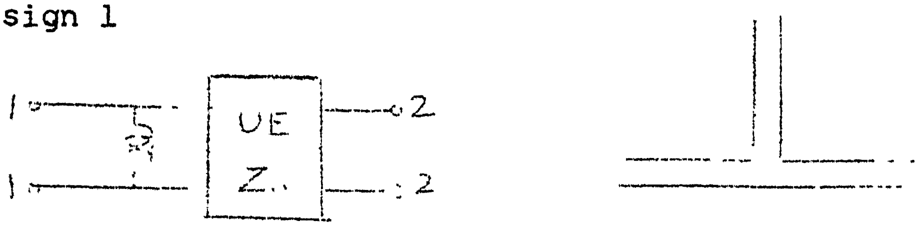
$$(1 - \lambda^2)^{-1/2} \begin{bmatrix} 1 & \lambda Z_o \\ \lambda / Z_o & 1 \end{bmatrix} \begin{bmatrix} 0 & jZ_o \\ 1/Z_o & 0 \end{bmatrix}$$

This is equivalent to a UE with characteristic impedance of Z_o cascaded with an ideal imaginary gyrator. In electrical systoms a gyrator signified an element which interchanges the through and across variable through transformation, Imaginary gyrator, in addition to the transformation of variables, give a constant delay of $\omega = \frac{\pi}{2}$. Such devices although available in microwave, have not yet been developed in fluidics, Hence there seems to be no way of extending the concept of frequency transformation (LP to HP and so on) to fluidic systems.

Apart from the transformation technique, direct synthesis procedure is also used for designing of HP, BP and BS microwave filters. One such procedure is the Richard's procedure discussed in Section 3.3. There is a variety of other methods, but they are all complex. However a number of such designs do exist which have been worked out through experience. We present here few of these designs which can be conveniently adopted for fluidic systems. These filters are designed based on image parameter theory.

i) HP Filters

Design 1



(a) HP Filter (b) Fluidic equivalent

Figure 5.21: HP fluidic filter design 1

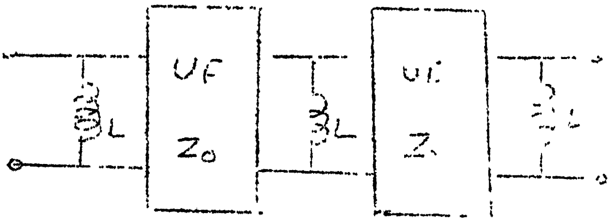
Z_{o1} and Z_{o2} are the image impedance from left and right respectively

$$\frac{Z_{o2}}{Z_{o1}} = \frac{1}{(1-\lambda_c^{\,2})^{1/2}}$$

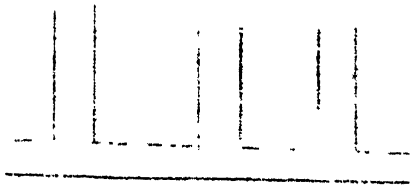
$$L = \frac{(1+\lambda_c^{\,2})^{1/2}}{c}$$

$$Z_o = \frac{1}{\lambda_c(1+\lambda_c^{\,2})^{1/2}}$$

Design 2



(a) HP Filter



(b) Fluidic equivalent

Figure 5.22: HP fluidic filter design 2

$$\frac{Z_{o2}}{Z_{o1}} = 1$$

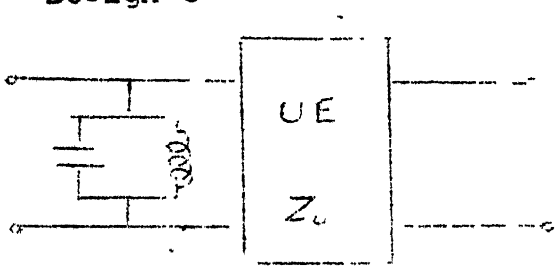
$$Z_o = 1$$

$$M = \left(1 + \frac{1}{\lambda_c^2} \right)^{1/2}$$

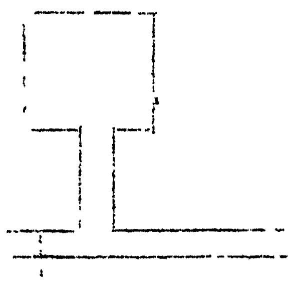
$$L = \frac{1 + m\lambda_c}{\lambda_c^2}$$

ii) BP Filters

Design 3



(a) BP Filter



(b) Fluidic equivalent

Figure 5.23: BP fluidic filter design 3

$$\Delta = \lambda_2 - \lambda_1$$

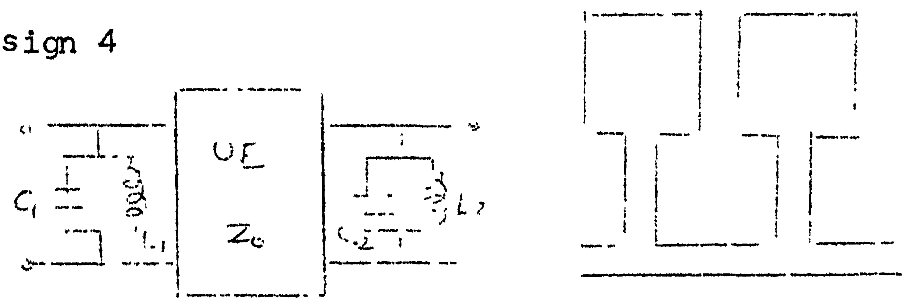
$$m = \left[\frac{(1+\lambda_2^{\ 2})}{(1+\lambda_1^{\ 2}} \right]^{1/2}$$

$$L = \frac{1}{m\Delta}$$

$$C = \frac{\Delta}{m\lambda^2}$$

$$\frac{Z_{o2}}{Z_{o1}} = 1$$

Design 4



(a) BP Filter (b) Fluidic equivalent

Figure 5.24: BP fluidic filter design 4

$$\frac{Z_{o2}}{Z_{o1}} = n^2$$

where $n = \dfrac{m(m^2-1)}{(m^2-a)}$ $\quad 1 \geq a \leq 0$

m & Δ as defined in earlier example

$$L_1 = \frac{\Delta m}{a\lambda_o^{\ 2}}$$

$$C_1 = \frac{1}{\Delta}$$

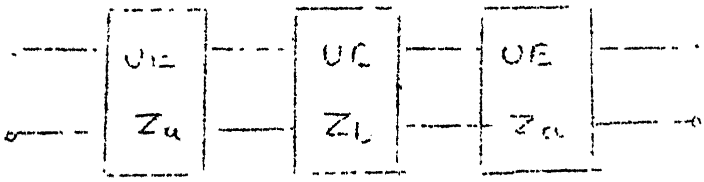
$$L_2 = \frac{Z_o(m^2-1)}{(1-a)}$$

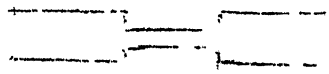
$$C_2 = \frac{1}{n^2 \Delta}$$

$$Z_o = \frac{\Delta m}{\lambda_o^{\,2}(m^2-a)}$$

iii) BS Filters

Design 5





(a) BS filter (b) Fluidic equivalent

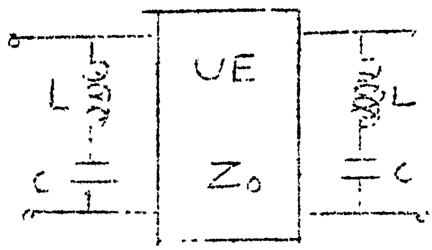
Figure 5.25:BS fluidic filter design 5

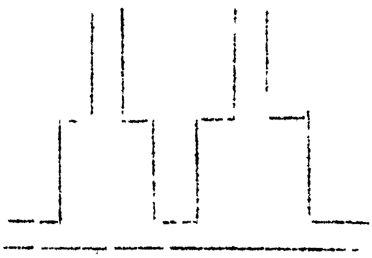
$$\frac{Z_{o2}}{Z_{o1}} = 1$$

$$Z_a = \frac{\lambda_1}{\lambda_2}\left[\frac{1+\lambda_2^{\,2}}{1-\lambda_1^{\,2}}\right]^{1/2}$$

$$Z_b = Z_a\left[\frac{1+\lambda_2^{\,2}}{1-\lambda_1^{\,2}}\right]^{1/2}$$

Design 6





(a) BS filter (b) Fluidic equivalent

Figure 5.26: BS fluidic filter design 6

$$\frac{Z_{o2}}{Z_{o1}} = 1$$

$$m = \frac{1 + \frac{1}{\sqrt{3}}}{[(1+\lambda_1^2)(1+\lambda_2^2)]^{1/2}}$$

$$L = \frac{m}{1-m}$$

$$C = \frac{1}{L\lambda_o^2}$$

$$Z_o = 1$$

Thus we see that Richard's transformation provides a very useful tool for synthesis of distributed parameter fluidic filters. We have made an attempt to attract the attention of people working in the field of fluidic acoustic and pneumatic systems to the usefulness of this concept. A vast literature exists on design of microwave filters using these techniques and it is expected that the difficulties encountered here can be overcome through further investigation.

SECTION 6

RESULTS OF PRACTICAL STUDY AND COMMENTS

In the preceeding section we discussed the procedure for design of fluidic filters using Richard's transformation. The work done in this regard is quite useful since this kind of approach, to the best of author's knowledge, is new to the field of fluidic filters. To make this proposition more attractive, a thorough study of practical behaviour of this type of fluidic filters is required.

We made an attempt to fabricate a prototype filter which could be tested for its performance with acoustic signals. But the work on the prototype filter could not take a satisfactory shape within the available time. A brief report on the efforts made in this direction is given below.

A filter with following specifications was selected for fabrication in the workshop.

A third order Butterworth filter with a cut off frequency of 1000 Hz; to be terminated in the source and load impedance of 1.6456×10^7.

For satisfactory results during testing, the filter should be terminated in a constant impedance which does not change with frequency. An infinite exponential horn is one

such arrangement which gives a fairly constant impedance. This impedance is given by the relation

$$Z = \frac{\rho a}{A_T}$$

where A_T is the area at the throat.

For the purpose of termination of the above filter, we will use an exponential horn at both source and load terminals.

Let the horns have a cross sectional area at the throat, A_T = 5x5 mm. The horns with this cross sectional area will give a constant input impedance of $1.6456 \times 10^7 \Omega$ as specified above. In relation to exponential horn, infinite length is an ideal situation which is not really possible in practice. A practical horn will have a length which gives a sufficiently large ratio of cross sectional areas between the mouth and the throat of the horn. With this fact in mind, we decided to choose the horns with a length of 50 cms.

The horns have to be of rectangular shape to match the ends of the filter. Each face of the rectangular horn will have a tapered shape, the dimensions of which can be found from the relation

$$A_x = A_T \, e^{mx}$$

where A_x is the area of cross section of the horn at a

distance x from the throat, and m is the coefficient of expansion taken equal to unity in this case [18].

The profile of the horn is given in Figure 6.1.

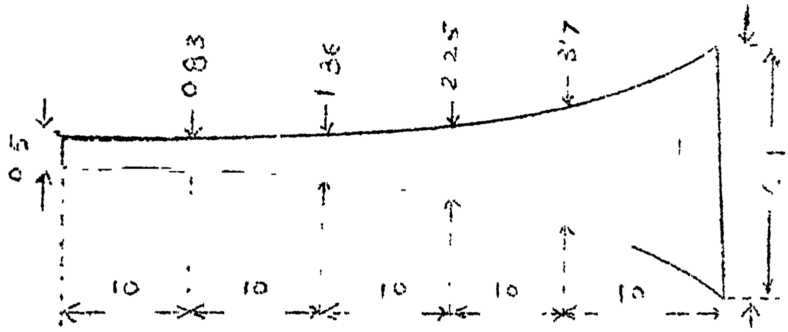


Figure 6.1: Profile of the horns. (The dimensions are given in cms).

The filter is synthesised from the table of fluidic filters given in Appendix 'G'.

The unit length which gives a transformed frequency of $\omega'_c = 1$ is 4.25 cms.

The element values are given below.

$C_1 = 0.5$

$Z_{C1} = 2$

$C_2 = 2$

$Z_{C2} = 2$

$C_3 = 0.5$

Considering a uniform depth of 5 mm, the width of various elements will be

$$C_1 = 2.5 \text{ mm}$$

$$Z_{C1} = 2.5 \text{ mm}$$

$$C_2 = 10.0 \text{ mm}$$

$$Z_{C2} = 2.5 \text{ mm}$$

$$C_3 = 2.5 \text{ mm}$$

The resulting filter configuration is given in Figure 6.2. The dimensions are given in mm.

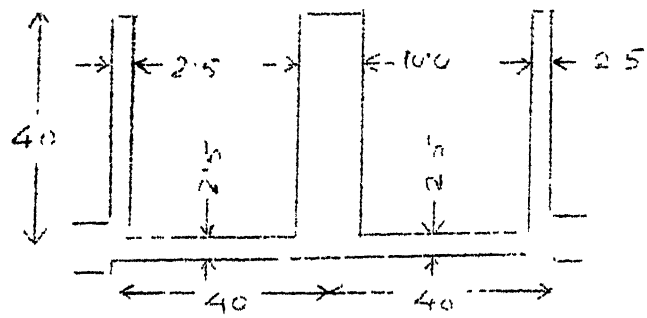
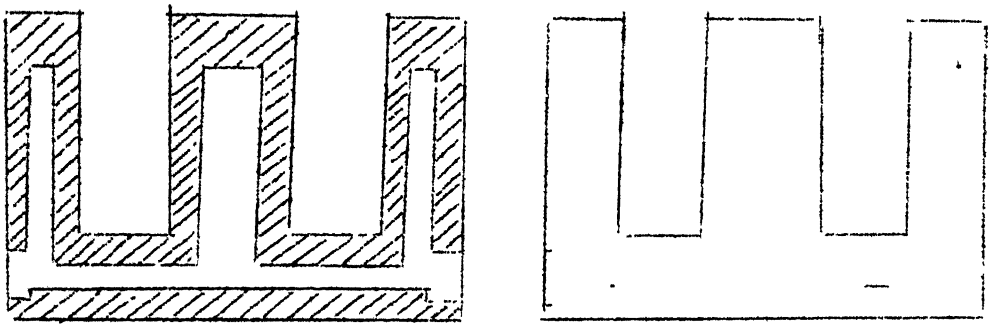


Figure 6.2: Proposed filter configuration

It was felt that the filter could not be fabricated by bending the metal sheet into a tubelar shape since the dimensions were very small for this purpose. Hence it was fabricated by hatching a solid brass plate on vertical milling machine. A plate of $\frac{1}{2}$'' thickness was used for this purpose. Another plate of $\frac{1}{4}$'' thickness was used to cover this piece. The plates were cut to shape to eliminate extra metal and give external appearance. The sketches of the plates are shown in Figure 6.3. The final shape of the filter block was quite satisfactory.



(a) Hatched plate (b) Cover plate

Figure 6.3: Fabrication of filter block

It was the fabrication of the horn that proved to be quite cumbersome. Brass sheet of 24 SWG was used for this purpose. The sheet was cut into 4 pieces of the profile given in Figure 6.1. The job of joining the plates to form a horn was quite tricky specially since the dimensions at the throat were very small. With great difficulty the two pieces could be joined together to give an angular shape but further brazing was not possible since the angular pieces could not be clamped properly.

An effort was made to make the inner core on which the plates could be tied and then brazed. But some provisionin problems caused an excessive delay thereafter, leaving no time for testing.

It can be still maintained that the proposed filter configuration is not very difficult to fabricate. It is the

fabrication of horn which proved to be rather difficult. The difficulties were encountered since it was the first job of its kind to be taken up in the workshop. Once the first piece or prototype is made, the other jobs can be made in quick succession.

Although practical studies could not take a constructive shape, the importance of the proposed filter design cannot be denied. There is evidence through experiments done by other scientists that the kind of elements we have used give a filtering effect. Therefore, we have a ground to believe that the filter schemes proposed by us will give satisfactory results in practice.

SECTION 7

CONCLUSION

Applications of fluidic filters are manifold in the field of pneumatic and acoustic systems in addition to those in fluidics itself. However, the developments in these areas have not reached a stage where their use would be very convenient and popular. We have made an attempt to improve the situation by exploiting some useful results of electrical filter theory in the design of fluidic filters. Our contribution in this regard can be specifically put in two categories.

i) For lumped parameter fluidic filters, we have applied modern filter design techniques. Modern filter theory allows us to design a filter to meet the given specifications exactly.

ii) For distributed parameter fluidic filters, we have used certain basic results of microwave filter theory to devise a new systematic synthesis procedure. The basis of this procedure is also the modern filter theory.

The synthesis procedure evolved here is fairly simple and straightforward, but certain points need to be examined through further investigation to make it more attractive for

usage in fluidic systems. Some important points relating to this suggestion are given below.

The first point is that we have used a simplified approach which will give a satisfactory result for acoustic type of signals where flow is basically of laminar type. This assumption will not always hold good, specially in systems involving flow of liquids where viscousity is considerably high. A suitably modified approach would have to be used for fluidic systems working under conditions of turbulance.

The second point emerges from the fact that the fluidic line model we have adopted is basically of lossless type. This is an ideal situation and in practice some losses will always be associated with the fluidic systems. The losses will be significant when the flow is turbulant or the cross sectional area of the tubes is very small. The conventional approach of electrical transmission line theory leads to a cumbersome treatment. An alternative model is required to be evolved to give a satisfactory result under these conditions.

The third problem associated with fluidic filters is that of frequency transformation. In case of electrical filters a low pass design can be conveniently converted to a high pass or a band pass filter through frequency transformation. This is not possible with fluidic filters as

discussed in Section 5.5. It is important to find a suitable remendy to make the design of HP, BP and BS filter also as simple and direct as it is in case of low pass filters.

It is expected that a suitable answer to the above questions can be found through further investigation in future. Further, it is hoped that the methods outlined here, when appropriatly moderated by practical experience would help in making the use of fluidic filters a popular and attractive proposition.

APPENDIX A

IMPORTANT FORMULAS

Given below is a list of important formulas used in this text.

1. Characteristic impedance, $Z_o \frac{\rho a}{A}$

2. Phase constant, $\beta = \frac{\omega}{a}$

3. Lumped fluidic inductance = $\frac{\rho}{A}$

4. Lumped fluidic capacitance = $\frac{A}{\rho a^2}$

5. Fluidic resistence of a circular tube $R = \frac{8\pi\mu}{A^2} \ell$

6. Fluidic resistence of a rectangular tube

$$R = \frac{12\mu}{A^2} \ell \ \sigma \left(1 + \frac{1}{\sigma^2}\right)$$

7. Input impedance of a fluidic line terminated in an impedance of Z_L

$$Z_{in} = Z_c = \frac{Z_L \cosh + Z_c \sinh}{Z_L \sinh + Z_c \cosh}$$

8. Input impedance of a closed ended fluidic line

$$Z_{op} = Z_c \frac{1}{\tan \Gamma \ell}$$

9. Input impedance of an open ended fluidic line

$$Z_{sh} = Z_c \tanh \Gamma \ell$$

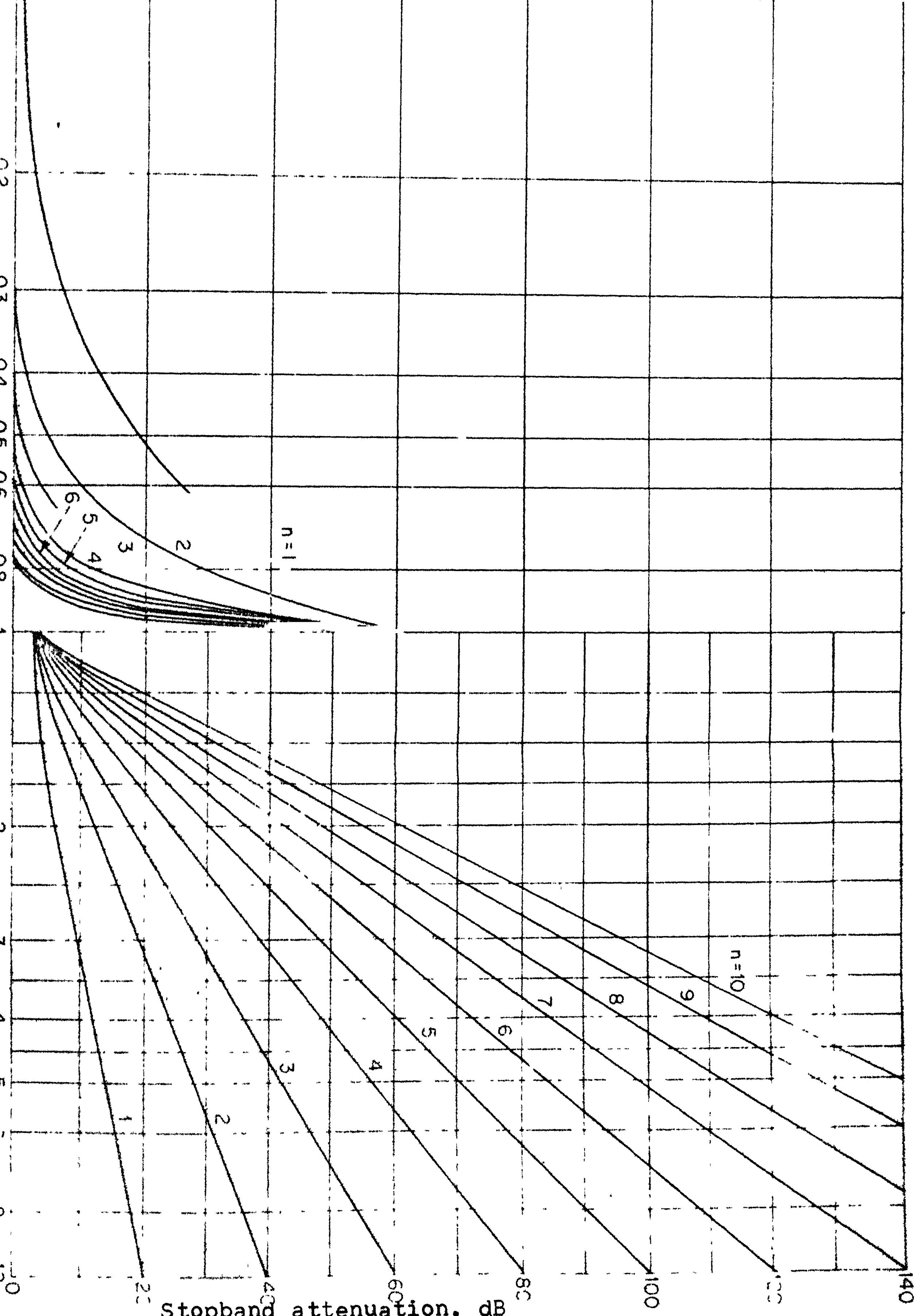


Figure B.1: Attenuation characteristics of Butterworth filters [9]

TABLE OF ELEMENT VALUES FOR BUTTERWORTH FILTERS

N	RL/RS	L1	C2	L3	C4	L5	C6	L7
2	1.000	1.414	1.414					
	1.111	1.035	1.835					
	1.250	0.848	2.121					
	1.429	0.697	2.438					
	1.667	0.566	2.828					
	2.000	0.448	3.346					
	2.500	0.342	4.095					
	3.333	0.245	5.312					
	5.000	0.156	7.707					
	10.00	0.074	14.81					
	100.0	1.414	0.707					
3	1.000	1.000	2.000	1.000				
	0.900	0.808	1.633	1.599				
	0.800	0.844	1.384	1.926				
	0.700	0.915	1.165	2.277				
	0.600	1.022	0.965	2.702				
	0.500	1.181	0.779	3.261				
	0.400	1.425	0.604	4.064				
	0.300	1.838	0.439	5.363				
	0.200	2.668	0.284	7.910				
	0.100	5.167	0.138	15.45				
	100.0	1.500	1.333	0.500				
4	1.000	0.765	1.848	1.848	0.765			
	1.111	0.466	1.592	1.744	1.469			
	1.250	0.388	1.695	1.511	1.810			
	1.429	0.325	1.862	1.291	2.175			
	1.667	0.269	2.103	1.082	2.613			
	2.000	0.217	2.452	0.883	3.187			
	2.500	0.169	2.986	0.691	4.009			
	3.333	0.124	3.883	0.507	5.338			
	5.000	0.080	5.683	0.331	7.939			
	10.00	0.039	11.09	0.162	15.64			
	100.0	1.531	1.577	1.082	0.383			
5	1.000	0.618	1.618	2.000	1.618	0.618		
	0.900	0.441	1.026	1.909	1.765	1.388		
	0.800	0.469	0.866	2.060	1.544	1.738		
	0.700	0.513	0.731	2.285	1.332	2.108		
	0.600	0.586	0.609	2.599	1.125	2.552		
	0.500	0.686	0.495	3.051	0.924	3.133		
	0.400	0.838	0.387	3.736	0.727	3.695		
	0.300	1.094	0.285	4.883	0.537	5.307		
	0.200	1.608	0.186	7.185	0.352	7.934		
	0.100	3.152	0.091	14.09	0.173	15.71		
	100.0	1.545	1.694	1.382	0.894	0.309		
6	1.000	0.518	1.414	1.932	1.932	1.414	0.518	
	1.111	0.289	1.040	1.322	2.054	1.744	1.335	
	1.250	0.244	1.116	1.126	2.239	1.549	1.688	
	1.429	0.207	1.236	0.957	2.499	1.346	2.061	
	1.667	0.173	1.407	0.801	2.858	1.143	2.509	
	2.000	0.141	1.653	0.654	3.369	0.942	3.094	
	2.500	0.111	2.027	0.514	4.140	0.745	3.930	
	3.333	0.082	2.656	0.378	5.432	0.552	5.280	
	5.000	0.053	3.917	0.248	8.020	0.363	7.922	
	10.00	0.026	7.705	0.122	15.78	0.179	15.73	
	100.0	1.553	1.759	1.553	1.201	0.758	0.259	
7	1.000	0.445	1.247	1.802	2.000	1.802	1.247	0.445
	0.900	0.298	0.711	1.404	1.489	2.125	1.727	1.296
	0.800	0.321	0.606	1.517	1.277	2.334	1.546	1.652
	0.700	0.357	0.515	1.688	1.091	2.618	1.349	2.028
	0.600	0.407	0.432	1.928	0.917	3.005	1.150	2.477
	0.500	0.479	0.354	2.273	0.751	3.553	0.951	3.064
	0.400	0.589	0.278	2.795	0.591	4.379	0.754	3.904
	0.300	0.774	0.205	3.670	0.437	5.761	0.560	5.258
	0.200	1.145	0.135	5.427	0.287	8.526	0.369	7.908
	0.100	2.257	0.066	10.70	0.142	16.82	0.182	15.74
	100.0	1.558	1.799	1.659	1.397	1.055	0.656	0.223

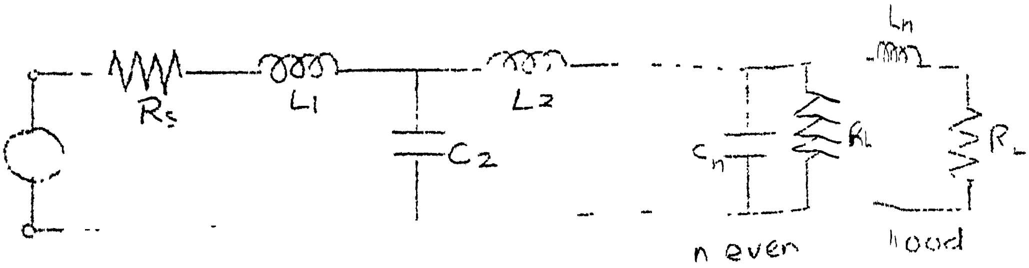


Figure C.1: The network congiruation for nth order Butterworth filter with voltage source at the input.

PROGRAM TO CONVERT A LOW PASS ELECTRICAL FILTER DESIGN
INTO A FLUIDIC FILTER DESIGN

```
PROGRAM FILTER(INPUT,OUTPUT);
   const R0=1.21E-03;
       A=340.0;
   var FR,X,M,I,J,K,N,UEF,UER:integer;
       D,L,WL,ZL,ZC0,TRFR:real;
       LORC,WUE,WSE,ZC:array[1..10]of real;
   procedure LTOC;
   begin ZC0:=ZC[I];
       ZC[I]:=ZC[I]+LORC[J];
       LORC[J]:=LORC[J]/ZC0/ZC[I]
   end;
    procedure CTOL;
   begin ZC0:=ZC[I];
       ZC[I]:=ZC[I]/(1+ZC[I]*LORC[J]);
       LORC[J]:=LORC[J]*ZC0*ZC[I]
   end;
    begin
        READ(N,D,WL,L,FR);
        TRFR:=sin(6.28E-03*FR*L/A);
        TRFR:=TRFR/sqrt(1-TRFR*TRFR);
        ZL:=1.0;
        for I:=1 to N do
           begin
              READ(LORC[I]);
              LORC[I]:=LORC[I]*ZL/TRFR
           end;
        UEF:=2*((N+2)DIV 4)-1;
        for I:=1 to N-1 do
              ZC[I]:=ZL;
        M:=UEF;
        K:=1;
        repeat
              J:=K;
              for I:=UEF downto M do
                begin
                   if M mod 2=1 then LTOC
                   else CTOL;
                   J:=J-1
                end;
              M:=M-1;
             K:=K+1;
         until M=0;
         M:=UEF+1;
         X:=N mod 2;
         K:=N;
         while M<N do
            begin
               J:=K;
               for I:=(UEF+1) to M do
                 begin
                    if X mod 2=1 then LTOC
                    else CTOL;
                    J:=J+1
                 end;
                    X:=X+1;
               M:=M+1;
              K:=K-1
            end;
         for I:=1 to N-1 do
            begin
               WUE[I]:=WL/ZC[I];
               WRITELN(WUE[I])
            end;
         for I:=1 to N do
            begin
               WSE[I]:=WL*LORC[I];
               WRITELN(WSE[I])
            end;
      end.
```

INPUT

N=order of filter
D=Uniform depth of the filter configuration
WL=Width of the load line
L=Unit lenth
FR=Cut off frequency

OUTPUT

WUE[I]=Width of the unit element
WSE[I]=Width of the stub line

This appendix gives the dimensions of the fluidic filters for various values of unit lengths.The specifications filter are given below.
Cutoff frequency=1000 hzs
Dimentions of the source and load lines=5*5 mms
uniform depth of filter=5 mm

WUE[I] and WSE[I] indicate the widths of unit elements and stub lines in mms respectively.

FLUIDIC FILTER DESIGN FOR L=30
WUE[1]= 3.8611848652
WUE[2]= 2.4520133137
WUE[3]= 1.5164989084
WUE[4]= 1.5164989084
WUE[5]= 2.4520133137
WUE[6]= 3.8611848652
WSE[1]= 1.1388151496
WSE[2]= 4.6395747065
WSE[3]= 1.1468110382E+01
WSE[4]= 1.6160701215E+01
WSE[5]= 1.1468110382E+01
WSE[6]= 4.6395747065
WSE[7]= 1.1388151496

FLUIDIC FILTER DESIGN FOR L=40
WUE[1]= 4.0093210339
WUE[2]= 2.8548648953
WUE[3]= 2.0258517563
WUE[4]= 2.0258517563
WUE[5]= 2.8548648953
WUE[6]= 4.0093210339
WSE[1]= 9.9067894220E-01
WSE[2]= 3.7860698401
WSE[3]= 8.1778333187
WSE[4]= 1.0977738052E+01
WSE[5]= 8.1778333187
WSE[6]= 3.7860698401
WSE[7]= 9.9067894220E-01

FLUIDIC FILTER DESIGN FOR L=50
WUE[1]= 4.1628807783
WUE[2]= 3.2385857105
WUE[3]= 2.5595630109
WUE[4]= 2.5595630109
WUE[5]= 3.2385857105
WUE[6]= 4.1628807783
WSE[1]= 8.3711920976E-01
WSE[2]= 3.0199519097
WSE[3]= 5.8949409723
WSE[4]= 7.5590082406
WSE[5]= 5.8949409723
WSE[6]= 3.0199519097
WSE[7]= 8.3711920976E-01

FLUIDIC FILTER DESIGN FOR L=60
WUE[1]= 4.3339210152
WUE[2]= 3.6393760442
WUE[3]= 3.1390307247
WUE[4]= 3.1390307247
WUE[5]= 3.6393760442
WUE[6]= 4.3339210152
WSE[1]= 6.6607900261E-01
WSE[2]= 2.2686224877
WSE[3]= 1.0620070695
WSE[4]= 4.9864255785
WSE[5]= 4.0620070695
WSE[6]= 2.2686224877
WSE[7]= 6.6607900261E-01

PROGRAM TO CONVERT THE TABLE OF ELECTRICAL FILTER
DESIGN INTO THE TABLE OF FLUIDIC FILTER DESIGN

```
PROGRAM CONVERT(INPUT,OUTPUT);
  var
      F,X,M,I,J,K,N,UEF,UER,INCR:integer;
      ZCO,RS:real;
      LORC,ZC:array[1..10]of real;
  procedure LTOC;
 begin ZCO:=ZC[I];
     ZC[I]:=ZC[I]+LORC[J];
     LORC[J]:=LORC[J]/ZCO/ZC[I]
 end;
  procedure CTOL;
 begin ZCO:=ZC[I];
     ZC[I]:=ZC[I]/(1+ZC[I]*LORC[J]);
     LORC[J]:=LORC[J]*ZCO*ZC[I]
 end;
  begin
      for N:=2 to 7 do
         begin
             READ(N);
             WRITE(N:2);
             for INCR:=1 to 11 do
               begin
                   READ(RS);
                   WRITE(RS:6:3);
            for I:=1 to N do
              begin
                  READ(LORC[I]);
              end;
            UEF:=2*((N+2)div 4)-1;
            for I:=1 to N-1 do
                  ZC[I]:=1;
            M:=UEF;
            K:=1;
            repeat
                  J:=K;
                  for I:=UEF downto M do
                    begin
                        if M mod 2=1 then LTOC
                        else CTOL;
                        J:=J-1
                    end;
                  M:=M-1;
                  K:=K+1;
            until M=0;
            M:=UEF+1;
            X:=N mod 2;
            K:=N;
            while M<N do
              begin
                  J:=K;
                  for I:=(UEF+1) to M do
                    begin
                        if X mod 2=1 then LTOC
                        else CTOL;
                        J:=J+1
                    end;
                  X:=X+1;
                  M:=M+1;
                  K:=K-1
              end;
            for F:=1 to N do
              begin
                  WRITE(LORC[F]:6:3);
                        if F<>N then WRITE(ZC[F]:6:3);
              end;
                  WRITELN;
                  if incr<>11 then WRITE(' ':2);
               end;;
         end;
  end.
```

N	RL/RS	C1	ZC2	C3	ZC4	C5
2	1.000	0.585	2.414	1.414		
	1.111	0.508	2.034	1.834		
	1.250	0.458	1.848	2.120		
	1.428	0.410	1.696	2.437		
	1.667	0.361	1.565	2.828		
	2.000	0.309	1.447	3.346		
	2.500	0.254	1.342	4.094		
	3.333	0.196	1.245	5.311		
	5.000	0.134	1.156	7.706		
	10.00	0.068	1.074	14.810		
	INF.	0.585	2.414	0.707		
3	1.000	0.500	2.000	2.000	2.000	0.50
	0.900	0.446	1.808	1.632	2.599	0.615
	0.800	0.457	1.843	1.383	2.925	0.65
	0.700	0.477	1.915	1.164	3.277	0.69
	0.600	0.505	2.022	0.964	3.701	0.72
	0.500	0.541	2.181	0.779	4.260	0.76
	0.400	0.587	2.425	0.604	5.064	0.80
	0.300	0.647	2.837	0.438	6.363	0.84
	0.200	0.727	3.667	0.284	8.910	0.88
	0.100	0.837	6.166	0.138	16.45	0.93
	INF.	0.600	2.500	1.332	1.500	0.33
4	1.000	0.433	1.765	1.848	2.414	1.35
	1.111	0.317	1.466	1.592	2.149	2.00
	1.250	0.279	1.388	1.695	1.866	2.27
	1.428	0.245	1.325	1.862	1.605	2.55
	1.667	0.211	1.269	2.102	1.358	2.87
	2.000	0.178	1.217	2.451	1.121	3.29
	2.500	0.144	1.168	2.986	0.890	3.88
	3.333	0.110	1.123	3.882	0.664	4.83
	5.000	0.074	1.079	5.683	0.442	6.68
	10.00	0.037	1.039	11.09	0.222	12.1
	INF.	0.604	2.531	1.577	1.805	0.82
5	1.000	0.381	1.618	1.617	2.447	1.82
	0.900	0.306	1.440	1.025	2.366	1.76
	0.800	0.319	1.468	0.866	2.583	1.52
	0.700	0.339	1.512	0.731	2.889	1.30
	0.600	0.369	1.586	0.609	3.309	1.104
	0.500	0.406	1.686	0.494	3.908	0.91
	0.400	0.455	1.837	0.386	4.799	0.73
	0.300	0.522	2.094	0.284	6.320	0.53
	0.200	0.616	2.608	0.185	9.340	0.35
	0.100	0.759	4.152	0.091	18.38	0.17
	INF.	0.607	2.545	1.694	1.985	1.15
6	1.000	0.202	1.254	0.779	1.781	1.65
	1.111	0.154	1.183	0.581	1.555	1.28
	1.250	0.140	1.163	0.554	1.559	1.31
	1.428	0.127	1.146	0.537	1.576	1.37
	1.667	0.113	1.128	0.523	1.601	1.45
	2.000	0.099	1.109	0.512	1.635	1.57
	2.500	0.083	1.090	0.504	1.678	1.75
	3.333	0.065	1.070	0.498	1.732	2.05
	5.000	0.045	1.047	0.494	1.799	2.66
	10.00	0.024	1.024	0.495	1.886	4.46
	INF.	0.274	1.378	1.239	2.709	1.65
7	1.000	0.190	1.235	0.717	1.693	1.50
	0.900	0.157	1.186	0.533	1.436	1.00
	0.800	0.163	1.195	0.524	1.391	0.91
	0.700	0.172	1.208	0.522	1.349	0.84
	0.600	0.183	1.224	0.524	1.307	0.78
	0.500	0.196	1.244	0.532	1.263	0.72
	0.400	0.212	1.270	0.548	1.216	0.65
	0.300	0.232	1.303	0.581	1.169	0.56
	0.200	0.258	1.348	0.651	1.133	0.46
	0.100	0.290	1.409	0.836	1.166	0.29
	INF.	0.274	1.378	1.241	2.722	1.71

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